MTH 380 Test 2 solutions August 8, 1995

1.) Use the **definition** of the derivative to find $f'(x)$ where $f(x) = x^2$.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} (2xh + h) = 2x$$

(2.-5.) Find the derivative of each of the following using the shortcut rules. Please make your work easy to follow for partial credit.

2.) $y = \sqrt{2}$ $y' = 0$

3.)

$$f(x) = x^4 + 3x^2 - \sqrt{x} = x^4 + 3x^2 - x^{1/2}$$

$$f'(x) = 4x^3 + 6x - \frac{1}{2}x^{-1/2} = 4x^3 + 6x - \frac{1}{2\sqrt{x}}$$

4.)

$$h(x) = \frac{1}{(x^3+5)^3} = (x^3+5)^{-3}$$

$$h'(x) = -3(x^3+5)^{-4} 3x^2 = \frac{-9x^2}{(x^3+5)^4}$$

5.)

$$g(x) = (x+2)^5(2x-1)$$

$$g'(x) = 5(x+2)^4(2x-1) + (x+2)^5 2 = (x=2)^4\left[5(2x-1) + 2(x+2)\right] = (x+2)^4(12x-1)$$

6.) Find the equation of the line tangent to the graph of $y = x^3 - 2x + 2$ at $(0,2)$.

$y' = 3x^2 - 2$ $y' = -2$ *at* $(0,2)$ $y = -2x + 2$

7.) Find the absolute maximum and minimum of $y = x^3 - 2x^2 + x + 4$ on the interval [0, 5] .

$y' = 3x^2 - 4x + 1 = (3x-1)(x-1)$ *critical numbers at* $x = \frac{1}{3}, 1$

so we have: $(0,4)$, $(\frac{1}{3}, 4.15)$, $(1,4)$, $(5,84)$

absolute max: 84 *absolute* min: 4

8.)

$$P(x) = \begin{cases} 3x - 0.0001x^3 & \text{when} \quad 0 < x \le 150 \\ 3.75x - 0.0001x^3 & \text{when} \quad x > 150 \end{cases}$$

A. Find the marginal price for the 50th, 100th, and 130th pairs of footies. Recall that the marginal price is the rate of change of price and rate of change is measured by slope.

$$P'(x)=\begin{cases}3-0.0003x^2 & if \quad 0<x<150\\ 3.75-0.0003x^2 & if \quad x>150\end{cases}$$ Just plug the numbers into P′(x)

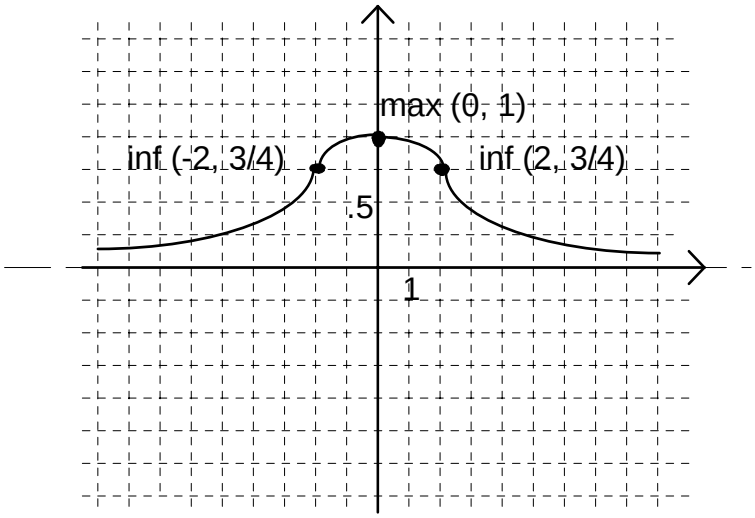
50th: 2.25 100th: 0 130th: -2.07

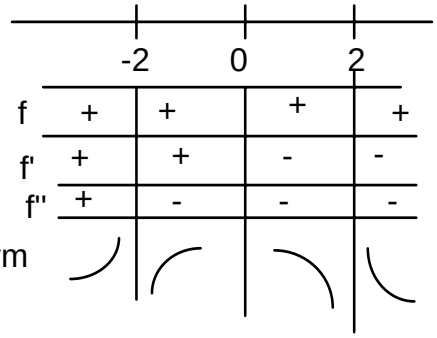
B. What is the average price for the first 50 pairs, 100 pairs, and 130 pairs of footies. Average equals total divided by number (P(x)/x)

50 pairs: 2.75 100 pairs: 2 130 pairs: 1.31

9.) Graph the following problem labeling all critical points, inflection points, asymptotes, x & y intercepts. Label the points with the x & y values in proper point notation. Label asymptotes with the equation of the line. Pay attention to concavity while graphing.

$$f(x)=\frac{12}{x^2+12}=12(x^2+12)^{-1}$$





$$f'(x)=-12\left(x^2+12\right)^{-2}2x=\frac{-24}{\left(x^2+12\right)^2}$$

$$f''(x)=\frac{-24\left(x^2+12\right)^2+24x2\left(x^2+12\right)2x}{\left(x^2+12\right)^4}=\frac{24\left(x^2+12\right)\left[-\left(x^2+12\right)+4x^2\right]}{\left(x^2+12\right)^4}=\frac{24\left(3x^2-12\right)}{\left(x^2+12\right)^3}=\frac{72(x+2)(x-$$