Final Exam Review Guide

The final exam will be broken into six sections as follows:

I. Evaluate limits graphically and algebraically; you will need to be able to evaluate both one and two sided limits. You will also need to be able to evaluate the value of a function at a point. (70 points)

II. Determine whether a function is continuous at a point, on an interval or on a domain. (40 points)

III. Differentiate all kinds of functions using all the techniques you've learned. (70 points)

IV. Determine whether a function has asymptotes and use its asymptotes and its derivatives as aids in graphing. (40 points)

V. Integrate polynomial, rational, and exponential functions. (40 points)

VI. Use differentiation and integration to solve applied problems. (40 points)

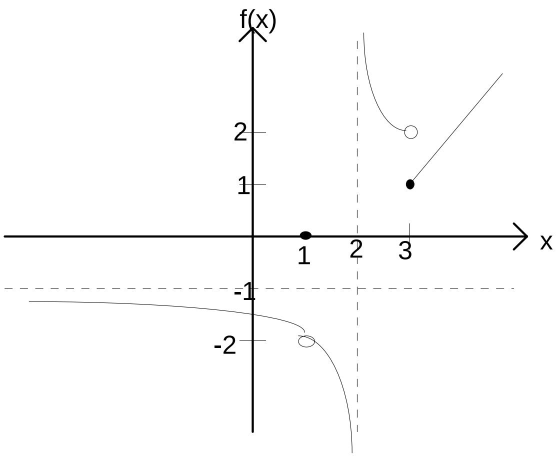
To practice problems of each type do the following.

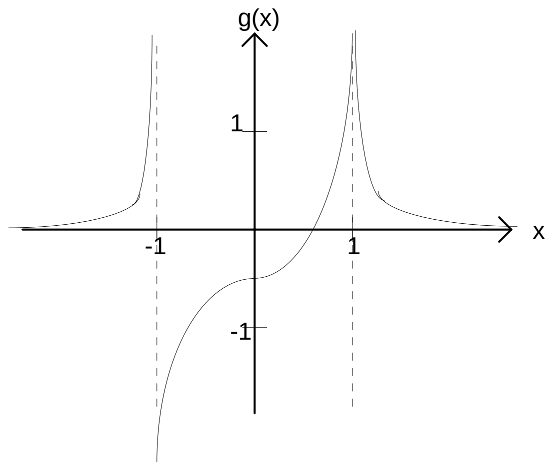
Note: the final exam will not be this long. If you understand and can do all the problems on this guide on your own you should do terrific on the final exam.

If there is an area where you feel you need more practice there are plenty of problems in the review sections of each chapter of your book that you could do for extra practice.

I.

1. Use the pictures below to evaluate the following: Use ∞ or –∞ where appropriate. Use d.n.e. for does not exist.





(a.) $\lim_{x \to -\infty} f(x)$

(b.) $\lim_{x \to 0} f(x)$

(c.) $\lim_{x \to 1} f(x)$

(d.) $\lim_{x \to 2-} f(x)$

(e.) $\lim_{x \to 2^+} f(x)$

(f.) $\lim_{x \to 3^-} f(x)$

(g.) $\lim_{x \to 3^+} f(x)$

(h.) $\lim_{x \to \infty} f(x)$

(j.) $\lim_{x \to 2} f(x)$

(k.) $\lim_{x \to 3} f(x)$

(l.) $f(1)$

(m.) $f(2)$

(n.) $f(3)$

(o.) $\lim_{x \to -\infty} g(x)$

(p.) $\lim_{x \to -1} g(x)$

(q.) $\lim_{x \to 0} g(x)$

(r.) $\lim_{x \to 1} g(x)$

(s.) $\lim_{x \to \infty} g(x)$

2.) Evaluate the following limits. Use ∞ or $-\infty$ where appropriate. Use d.n.e. for does not exist.

$$(a.)\quad \lim_{x\to 3} 5$$

$$(b.)\quad \lim_{x\to 1} \frac{x^2-1}{x-1}$$

$$(c.)\quad \lim_{x\to 1} \frac{x^2+1}{x-1}$$

$$(d.)\quad \lim_{x\to 2} \sqrt{x^3+8}$$

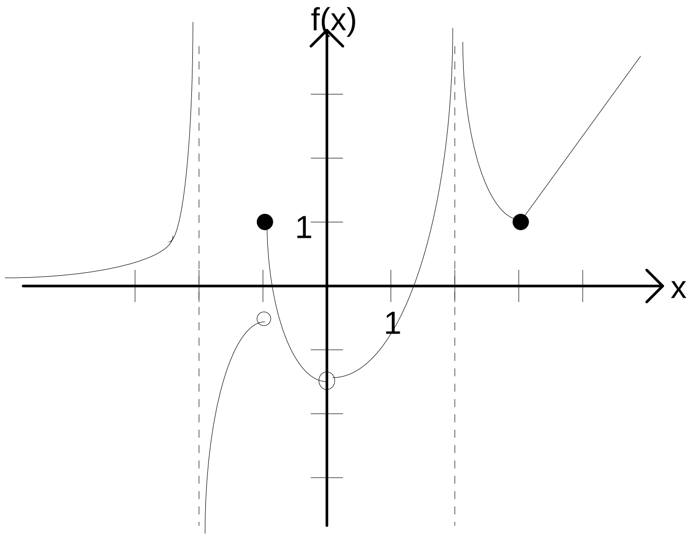
$$(e.)\quad \lim_{x\to 1} \frac{1}{(x-1)^2}$$

$$(f.)\quad \lim_{x\to\infty} \frac{3x^4+5x^2-2x+1}{-4x^3+3x^2-2x+1}$$

$$(g.)\quad \lim_{x\to\infty} \frac{3x^3+5x^2-2x+1}{-4x^3+3x^2-2x+1}$$

$$(h.)\quad \lim_{x\to\infty} \frac{3x^3+5x^2-2x+1}{-4x^4+3x^3-2x+1}$$

II. 1.) Refer to the following picture to answer the questions below.



(a.) Name all the x-values where f(x) is discontinuous.

(b.) Remove any removable discontinuities by altering the definition of f. (Fill in the blanks.)

f(x) is defined as in the picture above except f(____) = ________ .

2.) Determine where the following functions are continuous. Express your answers in interval notation.

$(a.)\quad f(x) = (5-x)^2$

$(b.)\quad g(x) = \frac{1}{5-x}$

$(c.)\quad h(x) = \sqrt{5-x}$

$(d.)\quad j(x) = \frac{1}{\sqrt{5-x}}$

$(e.)\quad k(x) = \frac{5-x}{x^2-25}$

$(f.)\quad l(x) = \frac{-1}{x+5}$

$(g.)\quad m(x) = \frac{x}{x^2-25}$

$$(h.)\quad n(x) = \begin{cases} x+2 & when \quad x < -2 \\ x^2+2x & when \quad -2 \le x < 5 \\ 5 & when \quad x \ge 5 \end{cases}$$

III.

1.) $f(x) = 5x^2 + 2x - 1$ *find* $f'(x)$

2.) $g(x) = \ln(3x^2 + 2)$ *find* $g'(x)$

3.) $h(x) = \sqrt{\ln(2x)}$ *find* $h'(x)$

4.) $f(x) = \log_3 \sqrt{x^3 - 1}$ *find* $f'(x)$

5.) $g(x) = \frac{3}{x^2+1}$ *find* $g'(x)$

6.) $h(x) = (x^2 + 2x - 5)^{10}$ *find* $h'(x)$

7.) $k(x) = 2xe^{5x^2+2x-1}$ *find* $k'(x)$

8.) $l(x) = \frac{2x^2+1}{3-x^2}$ *find* $l'(x)$

9.) $n(x) = 5^x$ *find* $n'(x)$

10.) $p(x) = (5)3^{x^2+2}$ *find* $p'(x)$

11.) $q(x) = x^2\sqrt{e^{2x} + x^2}$ *find* $q'(x)$

12.) $y = e^{3y+x} - x^2$ *find* $\frac{dy}{dx}$

13.) $xy = xy^2 + 2$ *find* $\frac{dy}{dx}$

IV.

1.) The graph of the function $F(x) = ax^3 + bx^2 + c$ contains critical points at (-2, 5) and (0, 1). Find the values of a, b, and c.

2.) The graph of the function $f(x) = ax^3 + bx^2$ contains an inflection point at (-1, 4). Find the values of a and b.

3.) Match each of the functions whose derivatives are given with one of the graphs below.

$(a.)\quad f'(x) = x(x+1)$

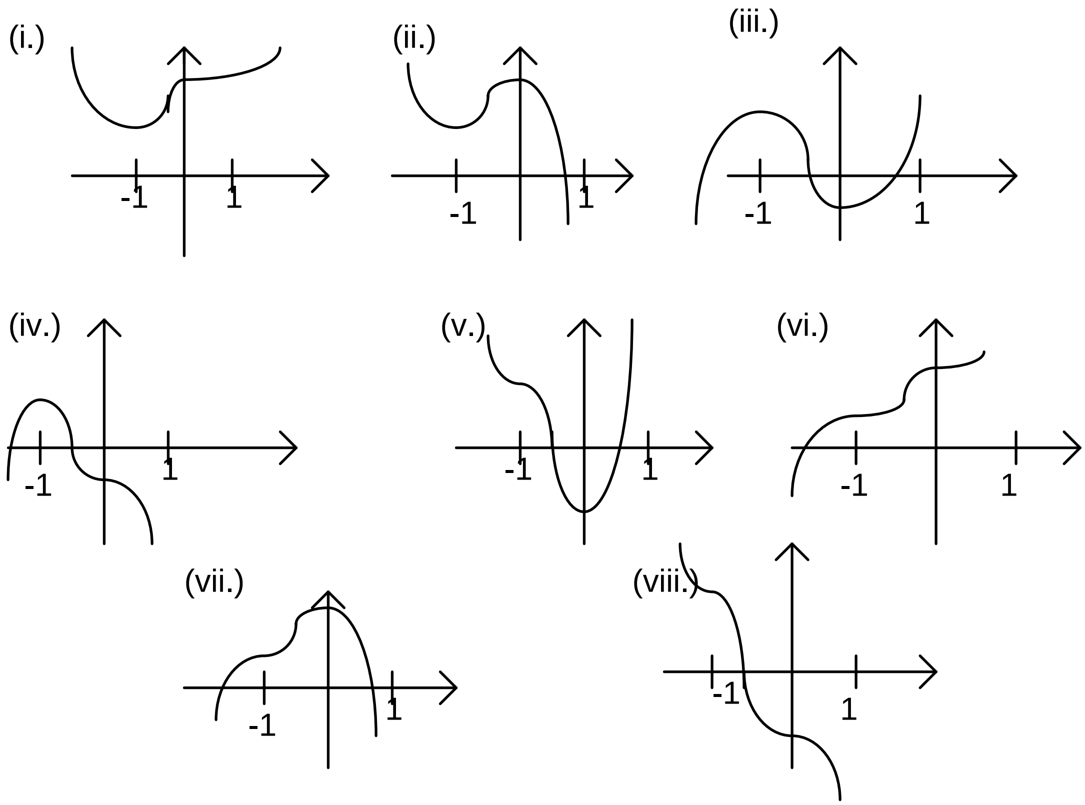
$(b.)\quad g'(x) = x^2(x+1)$

$(c.)\quad F'(x) = x(x+1)^2$

$(d.)\quad G'(x) = x^2(x+1)^2$

$(e.)\quad h'(x) = -x(x+1)$

$(f.)\quad H'(x) = -x(x+1)^2$

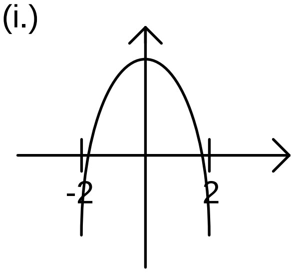


4.) Match each of the functions, whose second derivatives are given below with one of the graphs below.

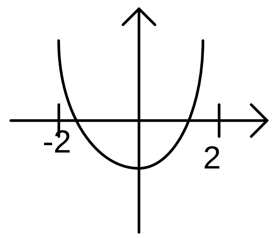
$(a.)\quad f''(x) = x(x-2)$ $\qquad$ $(c.)\quad g''(x) = x(x-2)^2$

$(b.)\quad F''(x) = x^2(x-2)$ $\qquad$ $(d.)\quad G''(x) = x^2(x-2)^2$

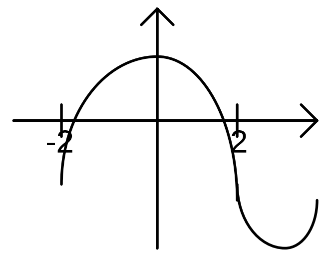
(i.)



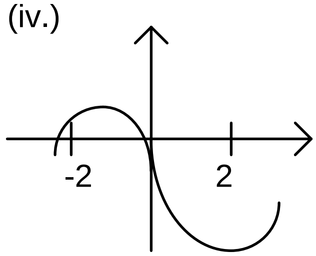
(ii.)



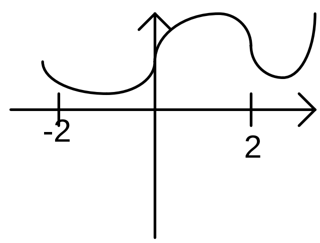
(iii.)



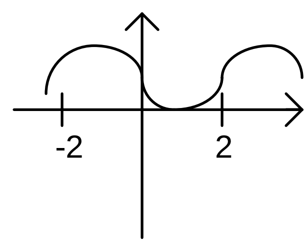
(iv.)



(v.)



(vi.)



5.) Graph the following functions. Label x and y intercepts, critical points, and inflection points with their coordinates. Label asymptotes with their equations. Pay attention to concavity and the general shape of the function but the only exact points you need are the ones named above.

(a.) $f(x) = 2x^2 - 4x - 3$

(b.) $g(x) = 4\sqrt{x} - x$

(c.) $h(x) = \dfrac{x^2}{x-2}$

(d.) $k(x) = \dfrac{2}{x^2+1}$

(e.) $F(x) = (x-1)^2(x-4)$

V. Evaluate the following:

1.) $\int dx$

2.) $\int \left(e^{2x} - x^{-1}\right)dx$

3.) $\int \left(x^3 + 2x\right)dx$

4.) $\int \frac{4x^3 + 5x^2 + 3}{2x^2}dx$

5.) $\int_0^9 x^{1/2}dx$

6.) Find *f*(*x*) when $f'(x) = \frac{2}{\sqrt[3]{x^2}}$ and *f*(8) = 14 .

VI. Solve the following story problems:

1.) The population of the world P(t) (in billions) can be approximated by the equation $P(t) = 5e^{0.002(t-1990)}$ where t is the year. Find the population and the rate of population growth in the year 2000.

2.) The revenue R(*x*) is related to the number of units sold *x*, by the equation R(*x*) = 10*x* - *x*ln*x* . How many units should be sold to maximize revenue?

3.) The marginal cost for a company that produces outboard motors has the form $C'(x) = 300 + 0.02x$. If the fixed costs are $10,000 find C(x), the cost function.

4.) The annual depreciation of a delivery van is described by the equation $f(t) = 2000 - 400t$ dollars/year where t is the time, in years, from the date of purchase. What is the total depreciation of the van from t = 0 to t=3?

5.) The state legislature is considering a bill that would impose a tax of t cents per dollar spent on restaurant food. The relationship between annual spending in restaurants and the tax is estimated by the equation: $A(t) = 216 - 2t^2 \quad 0 \le t \le 8$. Find the equation that describes the governments annual revenue as a function of t. What value of t maximizes the governments revenue?

6.) The relationship between the weekly cost $C(x)$ and x, the number of keyboards produced each week by the Musax Manufacturing Company, is given by the equation $C(x) = 2000 + 75x + \frac{50}{\sqrt{x}} \quad x \ge 1$. How fast are weekly cost changing when $x = 100$ if keyboard production is increasing at a rate of five units per week. (Hint: This is a related rates problem.)

7.) The annual revenue R(x) from the sales of portable television sets is given by the equation $R(x) = 200x - x^2$ where x equals the number of units sold.

(a.) Find the marginal revenue when 30 portable television sets are sold.

(b.) Find the change in revenue when the number of units changes from 30 to 31.

8.) Annual profits of a new computer software company are given by the equation $P(t) = -1 + 0.5t - 0.01t^2$ where P represents the annual profits in millions of dollars and t represents the time in years from when the company started.

(a.) Find the rate at which the company's profits are changing when t = 5.

(b.) Find the average change of profits from year 4 to year 6 .