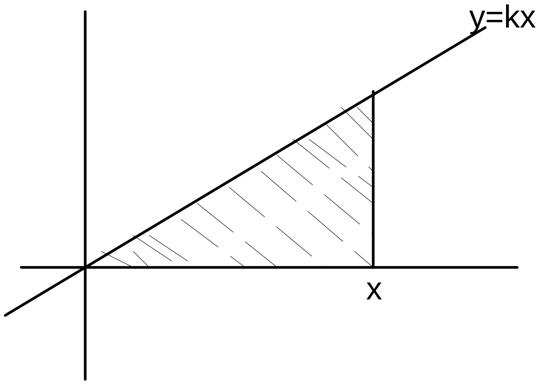
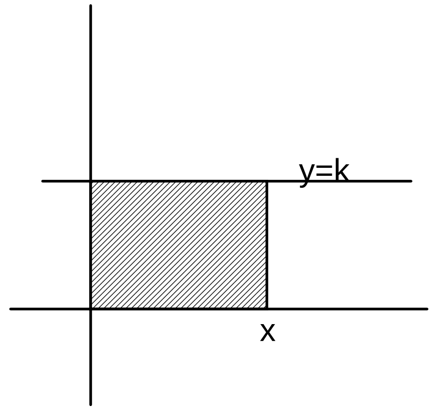
Definite Integrals and the Area Under the Curve related to sec 7-4 and 7-5

Let's look at the area A under two simple curves:





Area = ½ base × height = ½ xkx = ½kx^2

$A(x) = \frac{1}{2}kx^2$

$A'(x) = kx$

Area = kx

$A(x) = kx$

$A'(x) = k$

In general if A(x) represents the area under the curve $A'(x) = f(x)$ i.e. the rate of change of area equals the y value of the function at any given x. Thus

$A(x) = \int f(x)dx = F(x) + C$ if we start at the y-axis $A(0) = 0 = F(0) + C$

$-F(0) = C$

$A(x) = F(x) - F(0)$

If we want the area from a to b in the x axis then

$A(a) = F(a) - F(0) = \int_0^a f(x)dx$

$A(b) = F(b) - F(0) = \int_0^b f(x)dx$

so $A = A(b) - A(a) = F(b) - F(0) - (F(a) - F(0))$

$= F(b) - F(0) - F(a) + F(0)$

$= F(b) - F(a)$

$= \int_a^b f(x)dx$

This is called a definite integral and this relationship between area and antiderivatives is called the Fundamental Theorem of Calculus.