

Show **all of your work** on these pages. Transfer all relevant scratch work from your scrap paper to this test.

1. The earth's atmospheric pressure is modeled by assuming that the rate of change, $\frac{dp}{dh}$ of pressure p with respect to altitude h (above sea level) is proportional to p . Suppose that the pressure at sea level is 1000 millibars and that the pressure at an altitude of 25 km is 85 millibars.

(6 pts) a. Find the equation for $p(h)$ using the fact that $\frac{dp}{dh} = kp$ where k is a constant, $p = p_0$ when $h = 0$. Use the values above to determine p_0 and k .

(4 pts) b. What is the atmospheric pressure at $h = 50$ km?

(4 pts) c. At what altitude does the pressure equal 900 millibars?

2. Evaluate the following integrals **exactly** without a calculator. **Simplify your answer completely.**

(9 pts) a.
$$\int_0^{\sqrt{\ln 2}} x e^{3x^2} dx$$

(9 pts) b.
$$\int_1^8 \frac{3^{\log_2 x}}{x} dx$$

(9 pts) 3. Solve the following initial value problem:

$$\frac{dy}{dx} = 3 + \frac{1}{2x}, \quad x > 0 \quad \text{where } y = 2 \text{ when } x = 1.$$

(10 pts) 4. Differentiate $y = 5^{3x} + \ln\left(\frac{3x^2}{2^x+1}\right)$ (You do not need to simplify.)

(9 pts) 5. Use logarithmic differentiation to find and solve for $\frac{dy}{dx}$ in terms of x :

$$y = \frac{(x+2)(x^2-1)^2}{\sqrt{x+3}}$$

(10 pts) 6. Find the **exact** equation of the tangent line to the curve $y = (x + 2)^{x-1}$ at $x = 3$.
(Expect a messy answer.)

7. Let $f(x) = x^3 + x + 1$.

(6 pts.) a. Calculate $f'(x)$ and use it to show that f is one-to-one, thus showing f has an inverse function.

(4 pts.) b. Fill in the following charts:

x	$f(x)$
0	
1	
2	
3	

x	$f^{-1}(x)$
0	
-1	
-2	
-3	

(6 pts.) c. Evaluate $\frac{df^{-1}}{dx}$ at $(3, f^{-1}(3))$

(6 pts) 8. Evaluate:

a. $\lim_{x \rightarrow \infty} 0.9999^x$

b. $\lim_{x \rightarrow -\infty} 0.9999^x$

(8 pts) 9. Find the absolute maximum and minimum value of $y = \sqrt[3]{x} e^x$ on $[-1, 0]$.