Math 151 Final Exam May 5, 1998 Name ______________________________

Show any necessary work on these pages. Transfer any relevant work from scratch paper onto this test. For problems requiring some work, CIRCLE your answer.

1. Evaluate each of the following limits:

a) $\lim_{x \to 4} \frac{3}{(x-4)^2}$

Use the following definition of f to answer questions b through e.

$$f(x) = \begin{cases} x+4 & if \quad x<0 \\ 1 & if \quad x=0 \\ 4 & if \quad 0<x\le 2 \\ (x-2)^2 & if \quad x>2 \end{cases}$$

b) $\lim_{x \to 0} f(x)$

c) $\lim_{x \to 2^+} f(x)$

d) $\lim_{x \to 2^-} f(x)$

e) $\lim_{x \to 2} f(x)$

2. Let $g(x) = \begin{cases} 2x^3 + 3x^2 + m & , \quad x \le -1 \\ 2x^2 + m^2 x + 5 & , \quad x > -1 \end{cases}$

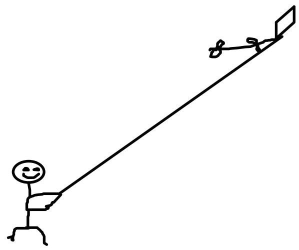
a) For what values of m is the function g continuous at x = -1? (Show work.)

b) For what values of m is the function g differentiable at x = -1? (Show work.)

3. Suppose $\frac{2x^3 - x^2 - 1}{4x^3 + 3x^2 + 2} < f(x) < \frac{x^2 + x + 1}{2x^2 - 5}$ for all $x > 2$. Find $\lim_{x \to \infty} f(x)$.

4. Find the equation of the tangent line to the graph $y = \frac{x+2}{x^2 - 3}$ at $x = 2$.

5. A child is flying a kite. If the kite is 90 feet above the child's hands and the wind is blowing it on a horizontal course at a rate of 5 *ft/sec*, how fast is the child paying out the cord when there is 150 feet of cord out? (Show work.)



6. The derivative of the continuous function *f* is:

$f' = x^{-1/3} - x^{2/3} = x^{-1/3}(1-x)$. Sketch the general shape of *f* and label the graph with the *x*-coordinates of all inflection points and local extrema. (Show work needed to arrive at your answer without a calculator.)

work	Show the intervals where *f* ' and *f* " are positive and negative. You may use a sign chart.

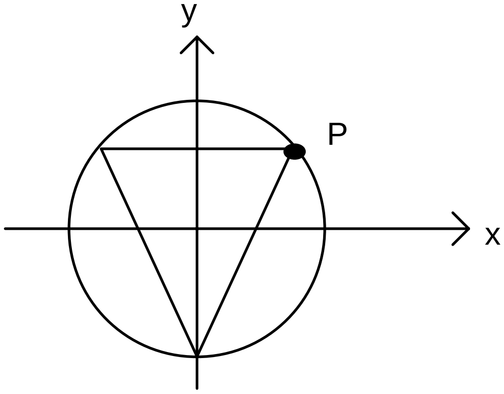
graph

7. For each of the following find $\frac{dy}{dx}$ without a calculator.

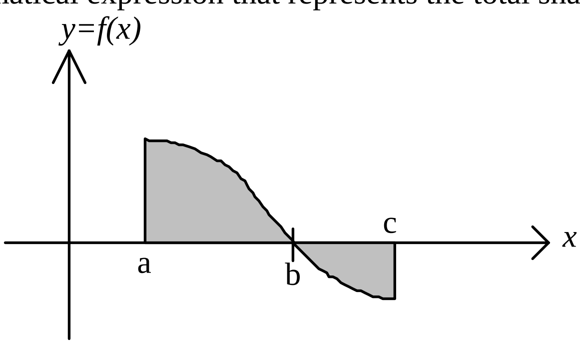
a) $y = x^{\cos x}$

b) $x^2 y + y^2 x = y$

8. Find the coordinates of the point P that will maximize the area of the isosceles triangle that is inscribed in a circle of radius one as pictured. (Show work.)



9. Let *f* represent a continuous function on [a, c] as pictured below. Write a mathematical expression that represents the total shaded area.



10. Which one of the following is the correct evaluation of $\int x^2 e^x dx$? Show work that leads to your conclusion. Circle the letter corresponding to the correct answer.

a) $x^2e^x - 2xe^x + 2e^x + c$

b) $\frac{1}{3}x^3e^x + x^2e^x + c$

c) $2xe^x + x^2e^x + c$

11. Evaluate $\int \left(e^{3x} + 3f(x)\right)dx$ if $\int f(x)dx = x^2\cos x + c$.

12. Evaluate $\dfrac{d}{dx}\displaystyle\int_{5}^{\sqrt{x}}\left(t^{2}+2t-1\right)^{2}dt$.

13. Solve the initial value problem: $\dfrac{dy}{dx}=\dfrac{\left(\sqrt{x}+1\right)^{3}}{2\sqrt{x}}+\dfrac{1}{x}$ where $y = 7$ when $x = 1$.

14. Suppose $f(x)$ has an inverse function and $f(3) = 4$, $f(4) = 6$, $f(5) = 3$, $f(6) = 5$, $f'(3) = -4$, $f'(4) = -2$, $f'(5) = -1$, and $f'(6) = -3$. Evaluate $\dfrac{d\left(f^{-1}(x)\right)}{dx}$ at $(5, f^{-1}(5))$.

15. Are you planning to come to class on Thursday to find out your grade and have some goodies? (Recall this is the only way to find out your grade before the report cards come in the mail.)