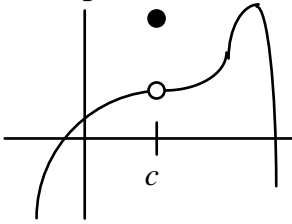


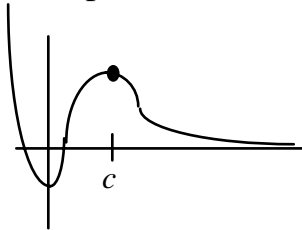
## Limits

Note: The limit of a function at a value of  $x$  has nothing to do with what the function equals at  $x$ . ( $\lim_{x \rightarrow c} f(x)$  doesn't necessarily equal  $f(c)$ .)

example 1



example 2



example 3

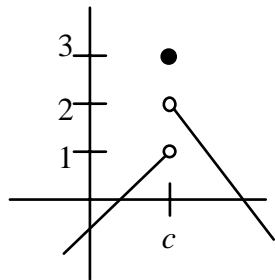
$$f(x) = \begin{cases} \frac{x^2 + x}{x} & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

Sometimes a function doesn't have a limit at a particular point. Example 4

$$f(x) = \frac{1}{x}.$$

Some functions approach a different value as  $x$  approaches a constant from the left (the negative (-)) than as  $x$  approaches that constant from the right (the positive (+)).

example 5



example 6

$$f(x) = \begin{cases} 2x^2 & \text{when } x > 1 \\ 2x - 1 & \text{when } x \leq 1 \end{cases}$$

If  $\lim_{x \rightarrow c^+} f(x) \neq \lim_{x \rightarrow c^-} f(x)$  we say  $\lim_{x \rightarrow c} f(x)$  does not exist.

When a function grows or shrinks without bound we say it goes towards plus or minus infinity. In example 4:  $\lim_{x \rightarrow 0^+} f(x) =$  and

$\lim_{x \rightarrow 0^-} f(x) =$ , even though  $\lim_{x \rightarrow 0} f(x)$  *d.n.e.* (does not exist).

We can also evaluate the limit of a function as  $x$  approaches  $\pm \infty$ .

In example 2:  $\lim_{x \rightarrow \infty} f(x) = 0$  and  $\lim_{x \rightarrow -\infty} f(x) = \infty$

In example 4:  $\lim_{x \rightarrow \infty} f(x) = 0$  and  $\lim_{x \rightarrow -\infty} f(x) = 0$