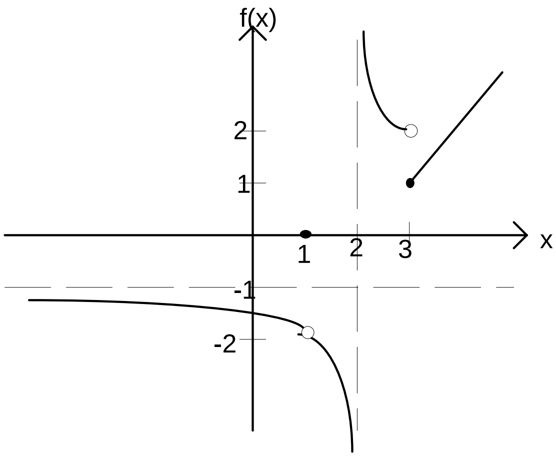
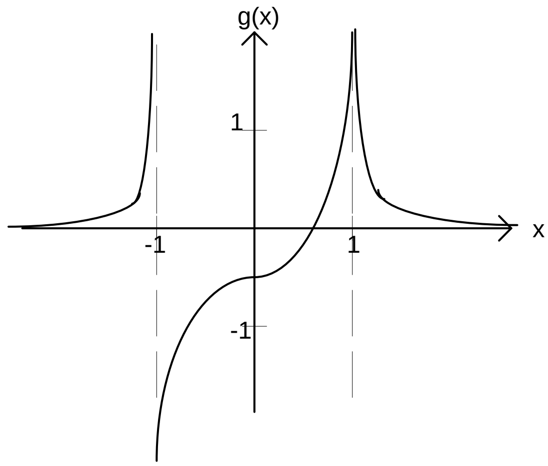
Final Exam Review Guide for Math 141

In addition to going over this guide you should make sure that you can do any problem that has been on a past test or quiz.

1. Use the pictures below to evaluate the following: Use ∞ or $-\infty$ where appropriate. Use d.n.e. for does not exist.





(a.) $\lim_{x\to-\infty} f(x)$

(b.) $\lim_{x\to 0} f(x)$

(c.) $\lim_{x\to 1} f(x)$

(d.) $\lim_{x\to 2-} f(x)$

(e.) $\lim_{x\to 2^+} f(x)$

(f.) $\lim_{x\to 3^-} f(x)$

(g.) $\lim_{x\to 3^+} f(x)$

(h.) $\lim_{x\to\infty} f(x)$

(j.) $\lim_{x\to 2} f(x)$

(k.) $\lim_{x\to 3} f(x)$

(l.) $f(1)$

(m.) $f(2)$

(n.) $f(3)$

(o.) $\lim_{x\to-\infty} g(x)$

(p.) $\lim_{x\to-1} g(x)$

(q.) $\lim_{x\to 0} g(x)$

(r.) $\lim_{x\to 1} g(x)$

(s.) $\lim_{x\to\infty} g(x)$

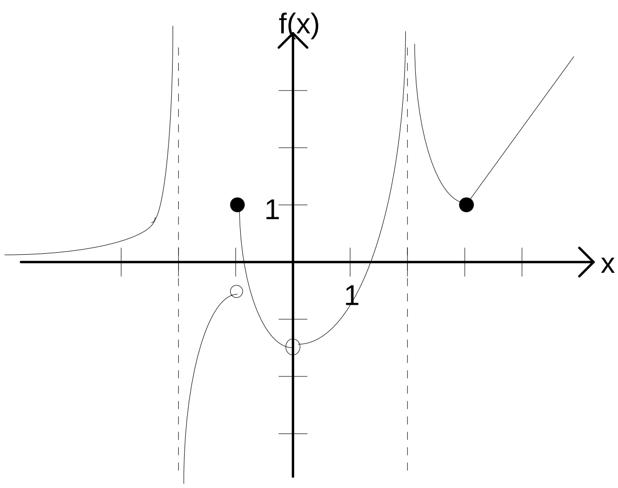
2.) Evaluate the following limits. Use ∞ or –∞ where appropriate. Use d.n.e. for does not exist.

$(a.)\quad \lim_{x\to 3} 5$ $\qquad (b.)\quad \lim_{x\to 1} \frac{x^2-1}{x-1}$

$(c.)\quad \lim_{x\to 1} \frac{x^2+1}{x-1}$ $\qquad (d.)\quad \lim_{x\to 2} \sqrt{x^3+8}$

$(e.)\quad \lim_{x\to 1} \frac{1}{(x-1)^2}$

II. 1.) Refer to the following picture to answer the questions below.



(a.) Name all the x-values where f(x) is discontinuous.

(b.) Remove any removable discontinuities by altering the definition of f. (Fill in the blanks.)

f(x) is defined as in the picture above except f(____) = ________ .

2.) Determine where the following functions are continuous. Express your answers in interval notation.

(a.) $f(x) = (5 - x)^2$

(b.) $g(x) = \frac{1}{5-x}$

(c.) $h(x) = \sqrt{5 - x}$

(d.) $j(x) = \frac{1}{\sqrt{5-x}}$

(e.) $k(x) = \frac{5-x}{x^2-25}$

(f.) $l(x) = \frac{-1}{x+5}$

(g.) $m(x) = \frac{x}{x^2-25}$

(h.) $$n(x) = \begin{cases} x + 2 & when \quad x < -2 \\ x^2 + 2x & when \quad -2 \le x < 5 \\ 5 & when \quad x \ge 5 \end{cases}$$

III. Derivatives

1.) $y = 5x^5 + 10x^2 - 3x^{-1}$ *find* y'

2.) $xy = xy^2 + 2$ *find* $\frac{dy}{dx}$

3.) $h(x) = \sqrt{\ln(3x+1)}$ *find* $h'(x)$

4.) $k(x) = (5x^2 - 3)^6$ *find* $k'(x)$

5.) $g(x) = x^2 e^{x^3}$ *find* $g'(x)$

6.) $h(x) = \frac{2x-5}{2x-3}$ *find* $h'(x)$

7.) $k(x) = \ln\sqrt{x^2 - 1}$ *find* $k'(x)$

IV.

1.) The graph of the function F(x) = $ax^3 + bx^2 + c$ contains critical points at (-2, 5) and (0, 1). Find the values of a, b, and c.

2.) The graph of the function f(x) = $ax^3 + bx^2$ contains an inflection point at (-1, 4). Find the values of a and b.

3.) Match each of the functions whose derivatives are given with one of the graphs below.

$(a.)\quad f'(x) = x(x+1)$

$(b.)\quad g'(x) = x^2(x+1)$

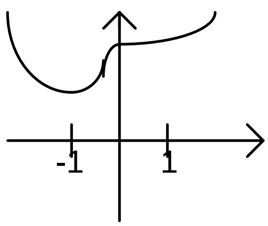
$(c.)\quad F'(x) = x(x+1)^2$

$(d.)\quad G'(x) = x^2(x+1)^2$

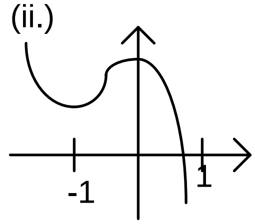
$(e.)\quad h'(x) = -x(x+1)$

$(f.)\quad H'(x) = -x(x+1)^2$

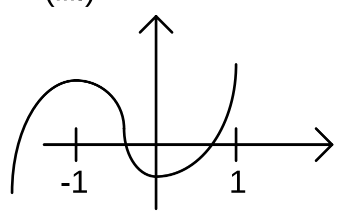
(i.)



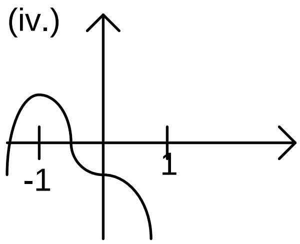
(ii.)



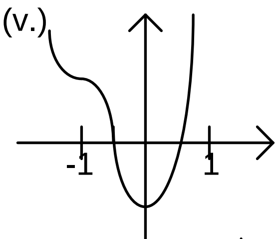
(iii.)



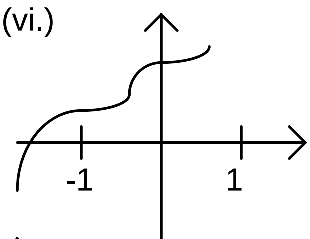
(iv.)



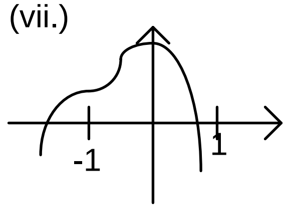
(v.)



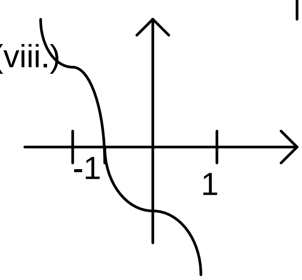
(vi.)



(vii.)



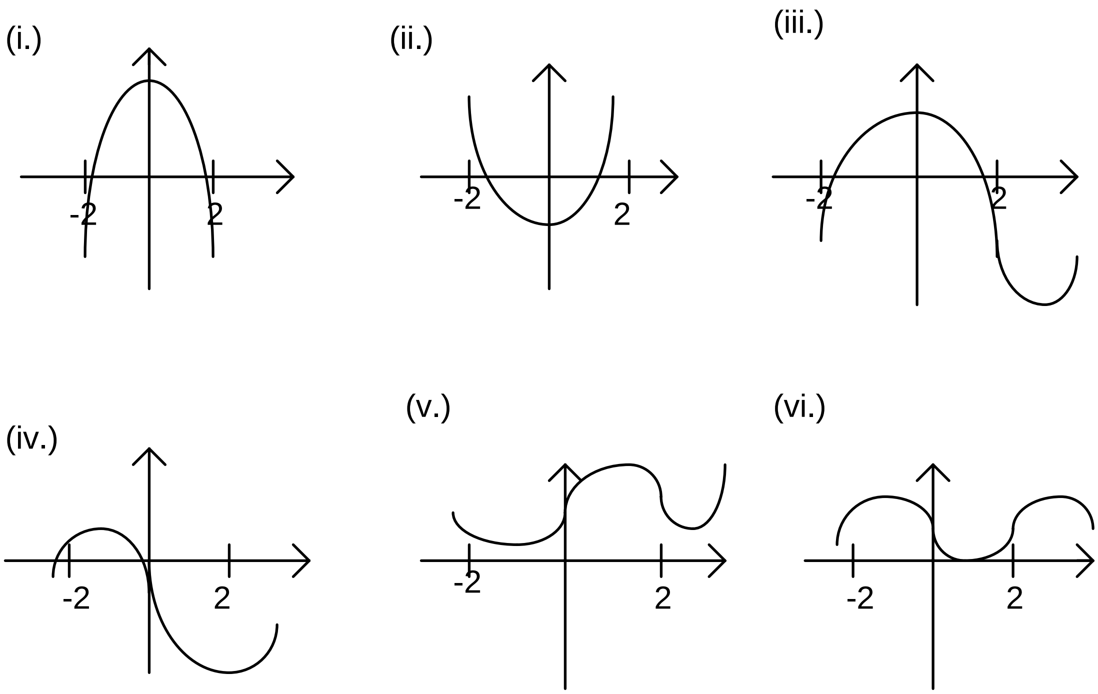
(viii.)



4.) Match each of the functions, whose second derivatives are given below with one of the graphs below.

$(a.)\quad f''(x) = x(x-2)$ $\qquad$ $(c.)\quad g''(x) = x(x-2)^2$

$(b.)\quad F''(x) = x^2(x-2)$ $\qquad$ $(d.)\quad G''(x) = x^2(x-2)^2$



5.) Find all critical points and inflection points for each of the following. For each critical point, describe the type it is.

(a.) $f(x) = 2x^2 - 4x - 3$

(b.) $g(x) = 4\sqrt{x} - x$

(c.) $h(x) = \dfrac{x^2}{x-2}$

(d.) $k(x) = \dfrac{2}{x^2+1}$

(e.) $F(x) = (x-1)^2(x-4)$

V. Evaluate the following:

1.) $\int dx$

2.) $\int \left(e^{2x} - x^{-1}\right)dx$

3.) $\int \left(x^3 + 2x\right)dx$

4.) $\int \frac{4x^3 + 5x^2 + 3}{2x^2}dx$

5.) $\int_0^9 x^{1/2}dx$

6.) Find $f(x)$ when $f'(x) = \frac{2}{\sqrt[3]{x^2}}$ and $f(8) = 14$.

VI. Solve the following story problems:

1.) The population of the world P(t) (in billions) can be approximated by the equation $P(t) = 5e^{0.002(t-1990)}$ where t is the year. Find the population and the rate of population growth in the year 2000.

2.) The revenue R(x) is related to the number of units sold x, by the equation R(x) = 10x - $x$ln$x$. How many units should be sold to maximize revenue?

3.) The marginal cost for a company that produces outboard motors has the form C′(x) = 300 + 0.02x . If the fixed costs are $10,000 find C(x), the cost function.

4.) The annual depreciation of a delivery van is described by the equation $f(t) = 2000 - 400t$ dollars/year where t is the time, in years, from the date of purchase. What is the total depreciation of the van from t = 0 to t=3?

5.) The state legislature is considering a bill that would impose a tax of t cents per dollar spent on restaurant food. The relationship between annual spending in restaurants and the tax is estimated by the equation: $A(t) = 216 - 2t^2 \quad 0 \le t \le 8$. Find the equation that describes the governments annual revenue as a function of t. What value of t maximizes the governments revenue?

6.) The relationship between the weekly cost $C(x)$ and x, the number of keyboards produced each week by the Musax Manufacturing Company, is given by the equation $C(x) = 2000 + 75x + \frac{50}{\sqrt{x}} \quad x \ge 1$. How fast are weekly cost changing when $x = 100$ if keyboard production is increasing at a rate of five units per week. (Hint: This is a related rates problem.)

7.) A 20 foot long ladder is leaning against a building. The foot of the ladder is slipping at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when the foot of the ladder is 5 feet from the wall?

8.) Annual profits of a new computer software company are given by the equation $P(t) = -1 + 0.5t - 0.01t^2$ where P represents the annual profits in millions of dollars and t represents the time in years from when the company started.

(a.) Find the rate at which the company's profits are changing when t = 5.

(b.) Find the average change of profits from year 4 to year 6 .