Math 122 Unit 5 Extra Assigned Problems

Last Update: Apr. 21, 2011

Problems for these exercises were pulled from Fall 2008, Fall 2010, and Summer 2010 tests. Look in the math 122 old test folder (http://laurashears.info/math122/unit5/oldTests/) if you would like to see more detailed solutions to these problems or more tests to practice.

Session 13, Part I: Introduction to Sequences and Series

1.) For each of the following, find the first five terms of the sequence.

A.) $a_1 = 2$, $a_2 = 0$; for $k \geq 2$, $a_{k+1} = a_k - a_{k-1}$	B.) $a_n = 3n - 8$
C.) $a_n = (-1)^{n-1}(2n - 6)$	D.) $a_n = 4a_{n-1} + 5$ where $a_1 = 3$

2.) Find the indicated term of the sequence: $a_n = (5n - 3)^2$; a_{12}

3.) Predict the general, or n^{th} term of the sequence: $\frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \ldots$

4.) Evaluate: $\sum_{i=2}^{4} 3^i$

5.) Rewrite with sigma notation: $8 + 11 + 14 + 17 + 20$

Session 13, Part II: Properties of Summation

1.) Use the properties of summations and the fact that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ to rewrite $\sum_{i=1}^{n} (ai^2 + bi + c)$ in terms of *a*, *b*, and *c*. Simplify your answer.

2.) Fill in the two blanks: $\sum_{k=1}^{30} [(k+1)^2 + c(k+1) + 9] = \sum_{k=\_\_}^{\_\_} [k^2 + ck + 9]$

3.) Evaluate: $\sum_{k=20}^{100} 2$

4.) Suppose $\sum_{k=1}^{20} 6a_k = 12$. Evaluate: $\sum_{k=1}^{20} 5a_k$

5.) Given that $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$. Evaluate $\sum_{k=1}^{18} (k+15)^3$

6.) Given $\sum_{k=1}^{5} c_k = 37$, evaluate $\sum_{k=1}^{5} (c_k - 1)$

Session 13, Part III: Arithmetic Sequences and Series

1.) What term of the arithmetic sequence is the given number? $\frac{7}{4}, \frac{9}{4}, \frac{11}{4}, \ldots$; $\frac{215}{4}$

2.) Find the indicated term: $a_1 = -1$, $d = 1/8$; a_{57}

3.) Find the common difference of the arithmetic sequence: $a_{50} = 203$ and $a_{76} = 307$.

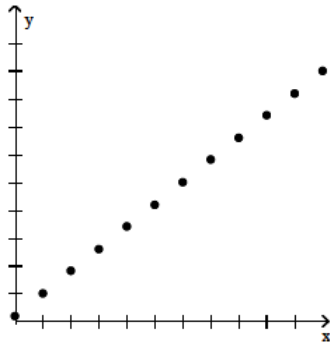
4.) Let $\{b_n\} = \{7, 4, 1, -2, -5, -8, ...\}$ Find the explicit and recursive formulas for b_n.

Session 14, Part I: Geometric Sequences and Series

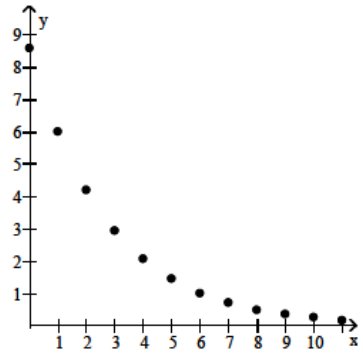
1.) Let $f(x) = 4(2^x)$, and consider the sequence f(1), f(2), f(3), ... Is this a geometric sequence, and if so, what is the common ratio?

2.) State whether the graph is that of an arithmetic sequence or a geometric sequence.

A.



B.



3.) Find the sum of the geometric series: $\sum_{i=1}^{30} 3(2)^i$

4.) Find the indicated term for the geometric sequence: $4, 4\sqrt{2}, 8, ...: 19^{th}$ term

5.) Evaluate $\sum_{k=1}^{15} 9\left(\frac{4}{3}\right)^k$ … a.) rounded to the thousandths place, b.) exactly.

6.) Gus decides to go bungee cord jumping using a cord that stretches 150 ft. He rebounds 65% of the distance fallen. How far has Gus traveled up and down by the time he comes to a rest? Round your answer to the thousandths place.

7.) Give the formula for the general term of the sequence: $\frac{16}{405}, \frac{8}{135}, \frac{4}{45}, \frac{2}{15}, ...$ where 16/405 is considered the first term.

8.) The geometric sequence b_n has $b_1 = 7$ and the common ratio $r = 2$. 117,440,512 is what term of the sequence?

Session 14, Part II: Combination and Factorial Notation

1.) Simplify: $\frac{(n+2)!}{(n-1)!}$

2.) Simplify: $\binom{s+1}{s-2}$

Session 15, Part I: Counting

1.) A ball is dropped from a height of 5.0 m. On each upward bounce the ball returns to 45 of its previous height. Find the total vertical distance the ball travels before coming to rest.

2.) In a test out of 25 questions a student must answer at least 23 questions correctly to get an A grade. How many ways can he take the test and receive an A grade?

3.) In a class of 15 students where grading is on a curve: 2 students will get an A, 3 students will get a B, 5 students will get a C, 3 students will get a D, and 2 students will get an F. How many ways can the grades be assigned? (Good thing we don't grade on a curve in this class.)

4.) A large family with 10 kids (6 boys (Alex, Cayman, David, Matt, Ralph, and Sid) and 4 girls (Fernanda, Gretchen, Jodie, and Sherry)) is planning some fun activities. For each of the following, tell the number of possible outcomes.

a) Some of the kids want to go to create a music group for an upcoming talent contest. How many different ways can they sign up as a group for the contest? Include the possibilities that nobody will sign up, everybody will sign up, or that someone will sign up by themselves.

b) Their neighbor, Paula invited them all to come and jump off her diving board into her pool, one at a time, but 4 of the kids need to stay home and help their parents with some chores first. How many ways can 6 kids out of the 10 get in line to jump in the pool?

5.) A number n has 2 prime factors: p and q. $n = p^3q^2$. How many factors (composite and prime) does n have? (Example: 6 is a composite factor of 12, 2 is a prime factor of 12, 12 is also a factor of itself.)

6.) How many ways can you make a 6 flag signal on a pole using 3 green flags, 2 blue flags, and 1 red flag?

7.) Thirteen students in the math club are going to send a team of 3 to compete in a contest against other schools. It was decided by the group that at least one of the two quickest problems solvers should be on the team. With this in mind, how many ways can a team be selected?

8.) How many ways can 3 Math and 5 Physics books be arranged on a shelf if all of the books from the same subject must be together?

Session 15, Part II: Binomial Theorem and Pascal's Triangle

1.) Write the indicated term of the binomial expansion: $(x + 3y)^{11}$, term containing x^7y^4

2.) Solve for x: $\sum_{i=0}^{7}\binom{7}{i}x^i 5^{7-i} = 2187$

3.) Expand and simplify: $(2x + 1)^5$.

4.) Evaluate the term containing x^6 in the expansion of $(3 - x^2)^{10}$.

Solutions:

Session 13, Part I: Introduction to Sequences and Series

1.) A.) 2, 0, -2, -2, 0 B.) -5, -2, 1, 4, 7 C.) -4, 2, 0, -2, 4
D.) 3, 17, 73, 297, 1193

2.) 3249 3.) $\frac{n+4}{n+5}$ 4.) $3^2 + 3^3 + 3^4 = 117$ 5.) $\sum_{i=1}^{5}(5+3i)$ (answer's vary)

Session 13, Part II: Properties of Summation

1.) $a\sum_{i=1}^{19} i^2 + b\sum_{i=1}^{19} i + \sum_{i=1}^{19} c = a\frac{(19)(20)(39)}{6} + b(20)\left(\frac{19}{2}\right) + 19c$
$= 2470a + 190b + 19c$

2.) blank below summation: 2, bland above summation: 31

3.) 162 4.) 10 5.) 300,321 6.) 32

Session 13, Part III: Arithmetic Sequences and Series

1.) 105 2.) 6 3.) 4

4.) recursive: $b_1 = 7$, $b_{n+1} = b_n - 3$; explicit: $b_n = 10 - 3n$

Session 14, Part I: Geometric Sequences and Series

1.) yes, 2 2.)A.) arithmetic B.) geometric 3.) 6,442,450,938 4) 2048

5.) a.) 2657.913 b.) $\frac{4,237,571,668}{1,594,323}$ 6.) 707.142 ft

7.) $\frac{16}{405}\left(\frac{3}{2}\right)^{n-1}$ 8.) 25th, used logarithms to figure this out

Session 14, Part II: Combination and Factorial Notation

1.) $n^3 + 3n^2 + 2n$

2.) $\frac{s^3 - s}{6}$

Session 15, Part I: Counting

1.) 45m 2.) 326 3.) 75,675,600 4a) 1024, 4b) 151,200
5.) 12 6.) 60 7.) 121 8.) 1440

Session 15, Part II: Binomial Theorem and Pascal's Triangle

1.) $26{,}730x^7y^4$ 2.) -2 3.) $32x^5 + 80x^4 + 80x^3 + 40x^2 + 10x + 1$
4.) $-262{,}440x^6$