Math 122 Unit 4 Extra Assigned Problems

Session 10, Part 1: Solving Systems of Equations with RREF Method

1.) Let $f(x) = ax^3 + bx^2 + cx + d$. Suppose the graph of f goes through the points (-3, 6), (-1, 1), (2, 5), and (4, -6). Find the equation for f. Show your setup and your results; then answer the question. Feel free to use your calculator's matrix feature for the work.

2.) Suppose a student uses matrices to solve a system of equations and after performing reduced row echelon reduction, the student gets the following matrix:

$\begin{bmatrix} 2 & 0 & 0 & -4 & 3 \\ 0 & 3 & 0 & 2 & 6 \\ 0 & 0 & -1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	Which of the following should the student conclude (circle the correct answer): i.) The system has no solutions. ii.) The system has one solution. iii.) The system had infinite solutions.

3.) Solve the following system of equations **by hand using matrices**. Label your operations for each new matrix. Obviously, you need to show your work on this one.

$$2x + 6y = -1$$
$$x - y = 3$$

4.) Each of the following matrices was set up to solve a system of equations for *x*, *y*, and *z* using reduced row echelon form by hand and was partially completed. Tell whether each system has zero solutions (inconsistent), one solution (independent), or infinite solutions (dependent).

A.) $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 2 & 0 \end{array}\right]$ ________________

B.) $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 2 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{array}\right]$ ________________

C.) $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 2 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{array}\right]$ ________________

5.) Vector **a** = -3**i** -8**j**, **b** = 13**i** + 4**j**, and **c** = -7**i** + 12**j**. Rewrite **c** in terms of **a** and **b** (as a linear combination of vectors **a** and **b**). You can use matrices and your calculator to figure out your answer.

Session 10, Part 2: Matrix Multiplication

1.) One of the following matrix multiplication problems has a solution. Circle the problem and name the dimensions of the product matrix (rows by columns).

a.) $\begin{bmatrix} 2 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

b.) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 & -2 \\ -1 & 1 & 3 \end{bmatrix}$

c.) $\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \end{bmatrix}$

d.) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 0 & -1 \\ 2 & -3 \end{bmatrix}$

2.) Solve for the constants a, b, and c in the following matrix multiplication problem.

$$\begin{bmatrix} a & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -2 & -3 \\ 1 & b \end{bmatrix}=\begin{bmatrix} -6 & -10 \\ c & 6 \end{bmatrix}$$

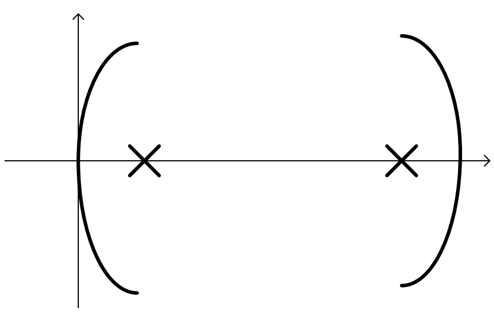
3.) Identify the unknowns in the following matrix multiplication problem.

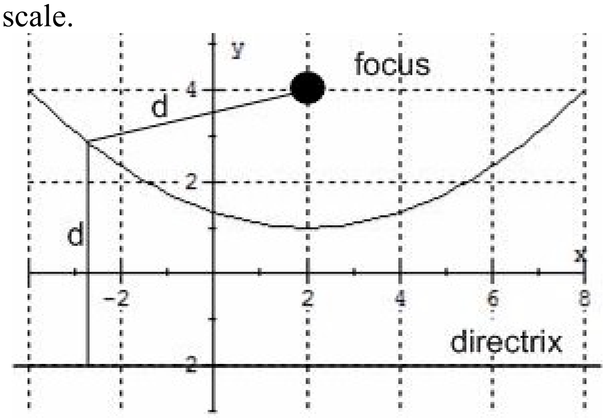
$$\begin{bmatrix} 1 & -1 \\ 2 & a \\ 3 & -2 \end{bmatrix}\times\begin{bmatrix} b & 2 & 5 & 4 \\ 3 & 1 & 3 & -2 \end{bmatrix}=\begin{bmatrix} 6 & d & f & g \\ 12 & 2 & 4 & 12 \\ c & e & 9 & 16 \end{bmatrix}$$

Session 11, Part 1: Parabolas

1.) Find the equation of the conic for which the set of all points in the plane have the same distance to the point (4, 6) as they have to the line $x = 2$.

2.) A children's museum has two parabolic shaped whisper dishes (image on left) across from each other in a large sometimes noisy room. The equation that describes the shape of a cross section of the dish is $16x = y^2$ where x and y are measured in feet. How far in front of the vertex of the dish should a mark be placed on the floor for the child to stand and hear the whispers from the other dish? The picture is not drawn to scale.

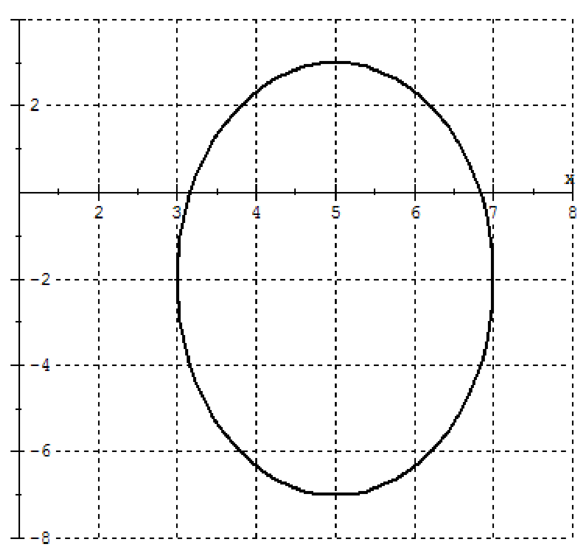




3.) Give the equation of the above conic (on the right) that is indicated in the image.

Session 11, Part 2: Ellipses

1.) Give the equation for the graphed conic on the right. Note that the y-axis is not visible.



2.) Find the equation for the conic that consists of set of all points in a plane for which the sum of the distances from two fixed points: (9, 3), (1, 3) is 12 units.

3.) Kidney stones can be broken up and removed underwater by using an elliptically shaped machine called a lithotripter, with an electrode at one focal point and the patient positioned so that the kidney stones are at the other focal point. The shockwaves emitted by the electrode break up the stones which can then pass through the patient's body in their urine.

A lithotripter has an inside depth 15 cm and an inside diameter of 18 cm. A cross section of the machine that cuts thru the middle forms half of an ellipse. How far from the vertex should a kidney stone be located?

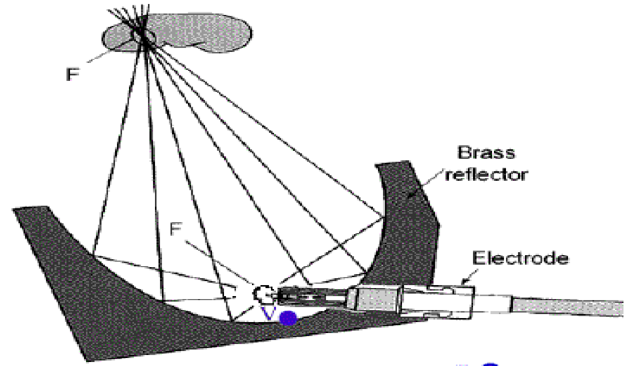
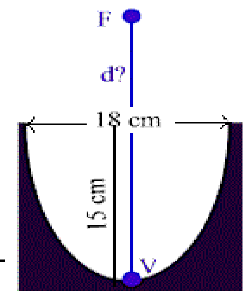


Figure 1 – Shockwave focus system.



4.) Find the equation for the set of all points in a plane where the sum of the distances from the points (7, 4) and (13, 4) is 9.

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Session 11, Part 3: Hyperbolas and All Conics

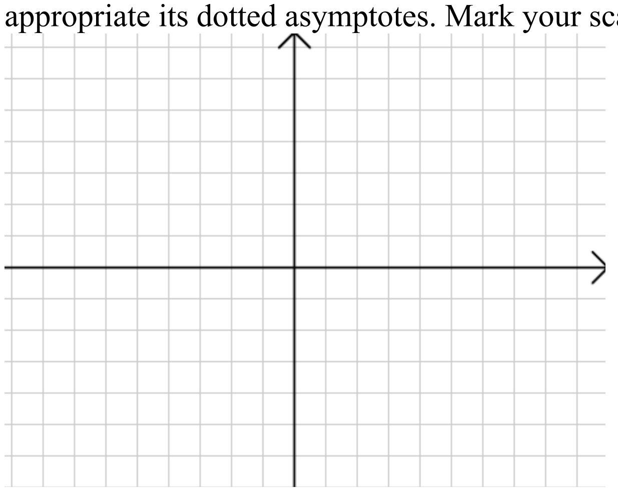
1.) Fill in the blanks with the missing word or phrase to complete the definitions.

parabola The set of all points in a plane for which the _____________ from a fixed point called a _______ and a fixed line called a _______________ are ___________.

ellipse The set of all points in a plane for which the _____________ of the ____________ from two fixed points called _________ is _____________.

hyperbola The set of all points in a plane for which the ___________ of the ____________ from two fixed points called _________ is ______________.

2.) Find the foci, vertices and center of the following conic and then draw the graph and if appropriate its dotted asymptotes. Mark your scales. $9x^2 + 18x - 64y^2 + 32y = 31$



center: ___________

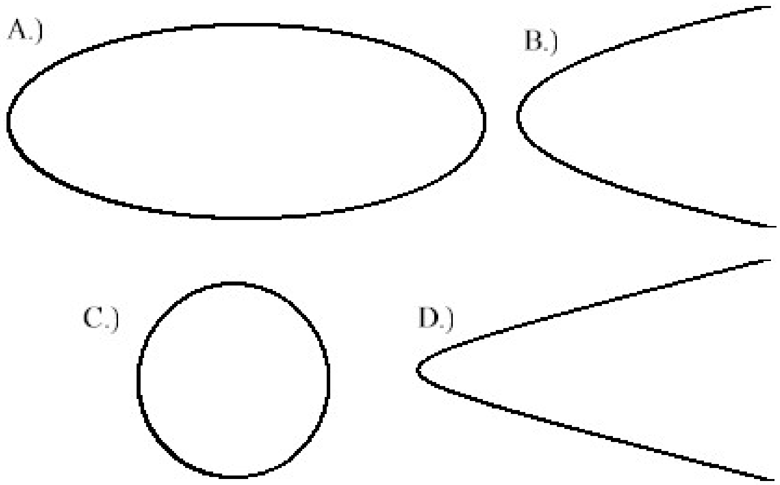
foci: ___________

vertices: __________

3.) Find the foci, vertices and center of the following conic and identify the conic

$$25x^2 - 9y^2 - 50x - 36y - 236 = 0$$

4.) Recall that eccentricity is defined to be *c*/*a* for both an ellipse and a hyperbola. Also, eccentricity could be thought of as the deviation from circular where the eccentricity of a circle is zero and of a parabola is one. With this in mind, name the order of the following images from smallest eccentricity to largest. Just write down their corresponding letters. The horizontal scale is equal to the vertical scale in each image.



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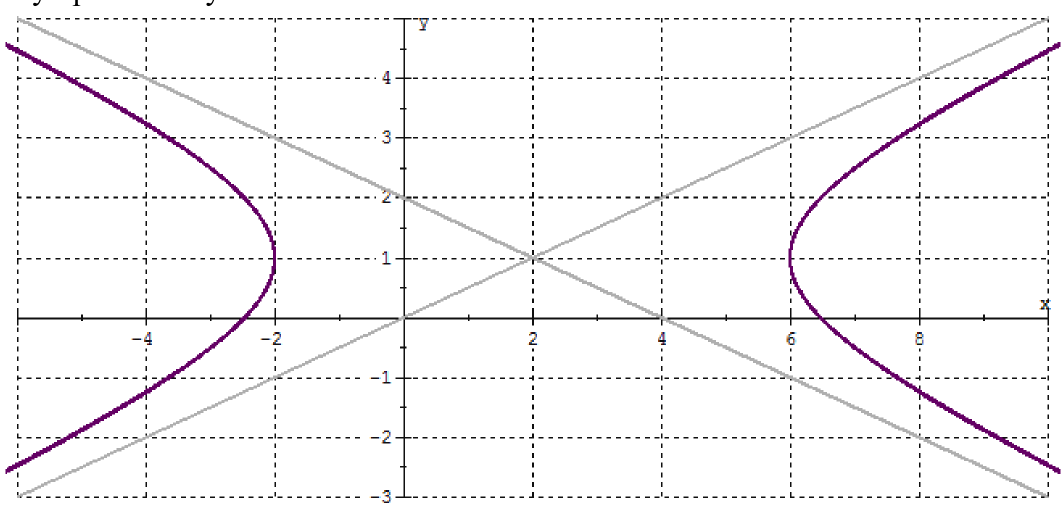
5.) Give the exact equation of the following conic in any form. The gray lines represent the asymptotes. Pay attention to the scale on each axis.



6.) Give the exact coordinates of the foci, and vertices for the following conic equation. Use proper coordinate notation.

$$\frac{(x+2)^2}{25}-\frac{(y-1)^2}{16}=1$$

7.) Recall that eccentricity is a measure of the deviation of a conic curve from circular. Two of the definitions of eccentricity are:

1. The ratio of the distances from any point on the conic: to the focus and to the directrix. In other words: for an arbitrary point *P*, on a conic with a focal point *F*, and a directrix, *D*, the eccentricity e = PF/PD.
2. The ratio of the distances from the center of the conic to the focus, and the center of the conic to the vertex. In other words: $e = c/a$, where c and a are the standard constants used in the conic equations.

With this in mind what would be the equation for the right directrix of the conic: $x^2/4 + y^2 = 1$?

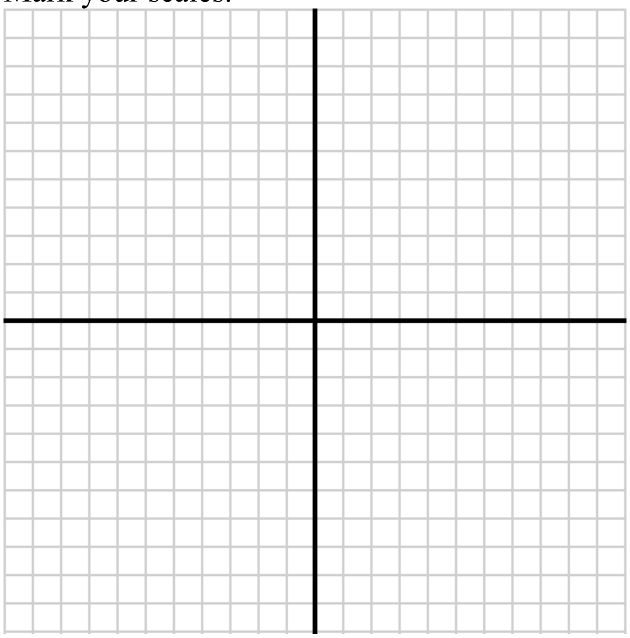
Session 12, Part 1: Parametric Equations

1.) Find a rectangular equation equivalent to the following set of parametric equations:
$x = \sec t$, $y = \tan t$, $0 < t < 2\pi$.

2.) Find a set of parametric equations equivalent to the rectangular equation: $x^2 + y^2 = 4$. Do not use $x(t) = t$. Do not use $y(t) = t$. Make sure you include restrictions on t.

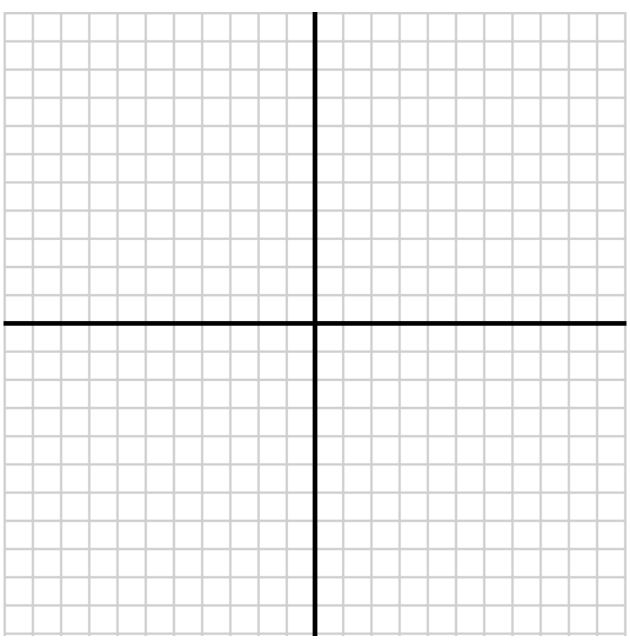
3.) Graph the plane curve given by the parametric equations:
Mark your scales.

$$x(t) = 2^{-t},$$
$$y(t) = -2^{t};$$
$$-3 \le t \le 3$$



4.) Come up with a set of parametric equations that describes the line segment that starts at (-2, -5) when $t = 0$ and ends at (2, 3) when $t = 4$.

5.) Sketch the graph of $x(t) = 6\tan^2(t) + 4$, $y(t) = 2\tan(t) + 2$ where t is in $[-\pi/2, \pi/2]$. Mark your scales.



6.) Match the rectangular equations to their equivalent set of parametric equations. Assume $t \in [0, 2\pi)$ for equations involving sine and cosine and $t \in (-\pi/2, \pi/2)$ for equations involving secant and tangent.

$$\frac{(x+4)^2}{4}+\frac{(y-1)^2}{16}=1$$ ____________

$$\frac{(x+4)^2}{4}-\frac{(y-1)^2}{16}=1$$ ____________

a) $x(t) = 2\cos t - 4,\ y(t) = 4\sin t + 1$

b) $x(t) = 2\cos t + 4,\ y(t) = 4\sin t - 1$

c) $x(t) = 2\sec t + 4,\ y(t) = 4\tan t - 1$

d) $x(t) = 2\sec t - 4,\ y(t) = 4\tan t + 1$

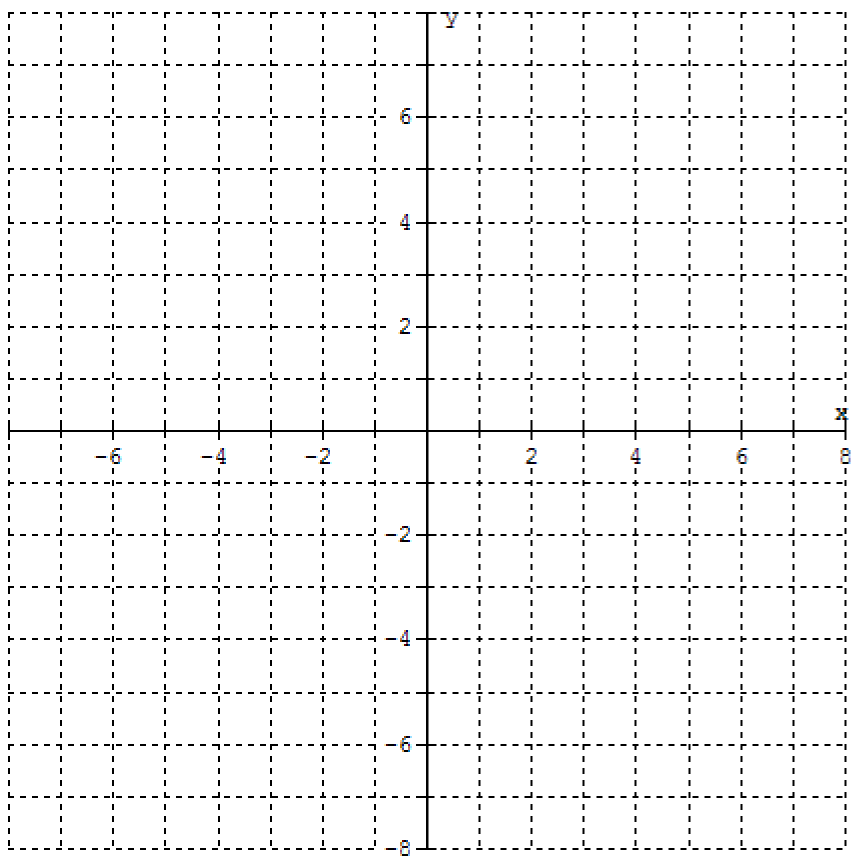
e) $x(t) = 2\tan t + 4,\ y(t) = 4\tan t - 1$

f) $x(t) = 2\tan t - 4,\ y(t) = 4\tan t + 1$

7.) Fill in the chart and graph the plane curve given by the parametric equations and give the equivalent rectangular equation.

$x = \sec^2 t - 4,\ y = \tan t + 1;\quad 0 \le t \le \pi$

t	x	y
0		
$\pi/6$		
$\pi/4$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$3\pi/4$		
$5\pi/6$		
π		



rectangular equation: ______________________________

Solutions:

Session 10, Part 1: Solving Systems of Equations with RREF Method

1.)

$$f(-3) = -27a + 9b - 3c + d = 6$$
$$f(-1) = -a + b - c + d = 1$$
$$f(2) = 8a + 4b + 2c + d = 5$$
$$f(4) = 64a + 16b + 4c + d = -6$$

$$\begin{bmatrix} -27 & 9 & -3 & 1 & 6 \\ -1 & 1 & -1 & 1 & 1 \\ 8 & 4 & 2 & 1 & 5 \\ 64 & 16 & 4 & 1 & -6 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & 0 & -32/105 \\ 0 & 1 & 0 & 0 & 11/70 \\ 0 & 0 & 1 & 0 & 439/210 \\ 0 & 0 & 0 & 1 & 92/35 \end{bmatrix}$$

$$f(x) = \frac{-32}{105}x^3 + \frac{11}{70}x^2 + \frac{439}{210}x + \frac{92}{35}$$

2.) You should have circled iii, the system has infinite solutions.

3.)

$$\begin{bmatrix} 2 & 6 & -1 \\ 1 & -1 & 3 \end{bmatrix} \Rightarrow \begin{matrix} R_1 \\ R_1 - 2R_2 \end{matrix} \begin{bmatrix} 2 & 6 & -1 \\ 0 & 8 & -7 \end{bmatrix}$$

$$\Rightarrow \begin{matrix} -3R_2 + 4R_1 \\ R_2 \end{matrix} \begin{bmatrix} 8 & 0 & 17 \\ 0 & 8 & -7 \end{bmatrix} \Rightarrow \begin{matrix} R_1/8 \\ R_2/8 \end{matrix} \begin{bmatrix} 1 & 0 & 17/8 \\ 0 & 1 & -7/8 \end{bmatrix}$$

$x = \frac{17}{8} = 2.125$ $\qquad$ $y = \frac{-7}{8} = -0.875$

4.) A.) 1 soln, independent B.) infin. solns, dependent C.) no solutions, inconsistent

5.) x<-3, -8> + y<13, 4> = <-7, 12>

<-3x + 13y, -8x + 4y> = <-7, 12>

$-3x+13y = -7$; $-8x + 4y = 12$

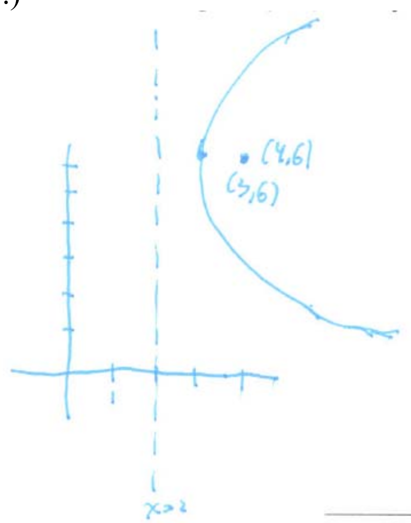
Use matrices rref or any method to solve the system and get $x = -2$, and $y = -1$,
so **c** = -2**a** - **b**

Session 10, Part 2: Matrix Multiplication

1.) part b: 2 × 3 $\qquad$ 2.) a = 4, b = 1, c = 5 $\qquad$ 3.) a= -2, b= 9, c = 21, d=1, e=4, f=2, g=6

Session 11, Part 1: Parabolas

1.)



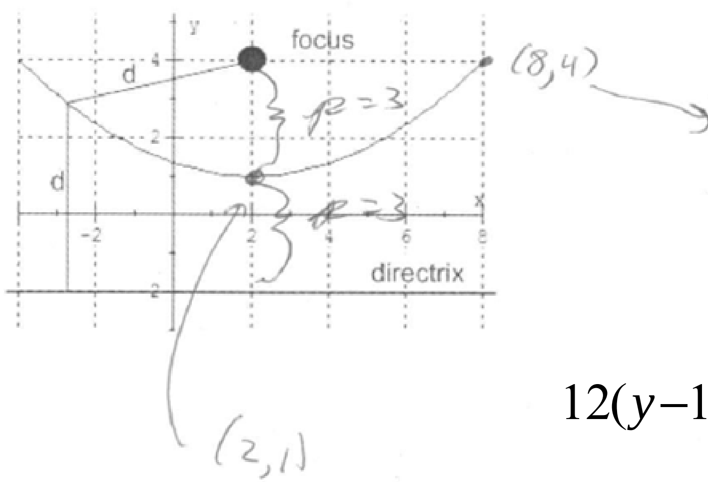
$p = 1$

$4p(x-3) = (y-6)^2$

$4(x-3) = (y-6)^2$

2.) $4p$ = 16 ft, so the mark should be placed 4 ft in front of the vertex.

3.)



Check

$12(4-1) = (8-2)^2$

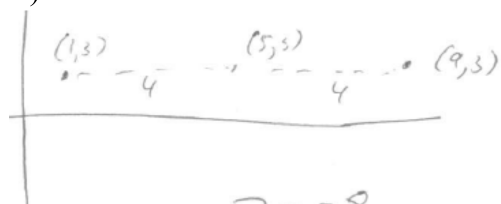
$36 = 36$ ✓

$$12(y-1) = (x-2)^2$$

Session 11, Part 2: Ellipses

1.) $\dfrac{(x-5)^2}{4} + \dfrac{(y+2)^2}{25} = 1$

2.)



$\dfrac{(x-5)^2}{36} + \dfrac{(y-3)^2}{20} = 1$

$2c = 8$

$c = 4$

$2a = 12$

$a = 6$

3.)

$a = 15$, $b = 9$

$c^2 = a^2 - b^2$

$c^2 = 15^2 - 9^2 = 144$

$c = 12$

$a + c = 15 + 12 = $ __27 cm__

4.) $\dfrac{(x-10)^2}{\left(\dfrac{81}{4}\right)} + \dfrac{(y-4)^2}{\left(\dfrac{45}{4}\right)} = 1$

Session 11, Part 3: Hyperbolas

1.) **parabola** The set of all points in a plane for which the _distances_ from a fixed point called a _focus_ and a fixed line called a _directrix_ are _equal_.

ellipse The set of all points in a plane for which the _sum_ of the _distances_ from two fixed points called _foci_ is _constant_.

hyperbola The set of all points in a plane for which the _difference_ of the _distances_ from two fixed points called _foci_ is _constant_.

2.)

$9(x^2 + 2x + 1) - 64(y^2 - \frac{1}{2}y + \frac{1}{16}) = 31 + 9 - 4$

$9(x+1)^2 - 64(y - \frac{1}{4})^2 = 36$

$\dfrac{(x+1)^2}{4} - \dfrac{16(y-\frac{1}{4})^2}{9} = 1$

$\dfrac{(x+1)^2}{4} - \dfrac{(y-\frac{1}{4})^2}{\frac{9}{16}} = 1$

$\dfrac{(x+1)^2}{2^2} - \dfrac{(y-\frac{1}{4})^2}{(\frac{3}{4})^2} = 1$

$c^2 = 4 + \frac{9}{16} = \frac{64+9}{16}$

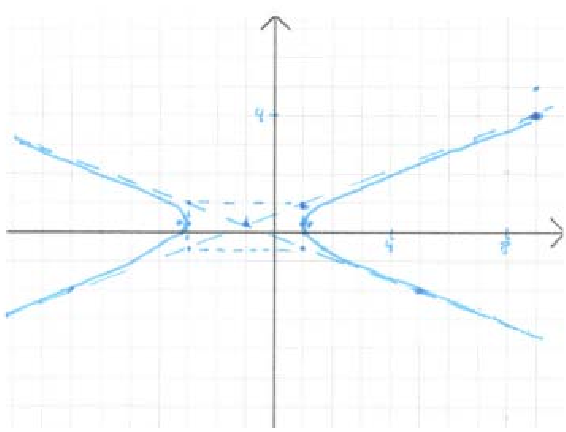
$c^2 = \frac{73}{16}$

$c = \frac{\sqrt{73}}{4}$

center: $(-1, \frac{1}{4})$

foci: $(-1 \pm \frac{\sqrt{73}}{4}, \frac{1}{4})$

vertices: $(1, \frac{1}{4})$, $(-3, \frac{1}{4})$



3.)

$25(x^2-2x+1)-9(y^2+4y+4)=236+25-36$

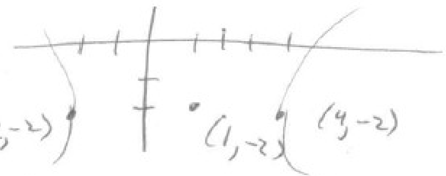
$25(x-1)^2-9(y+2)^2=225$

$\frac{(x-1)^2}{9}-\frac{(y+2)^2}{25}=1$

center: $(1,-2)$

$(-4.83,-2)$, $(6.83,-2)$

foci: $(1\pm\sqrt{34},-2)$



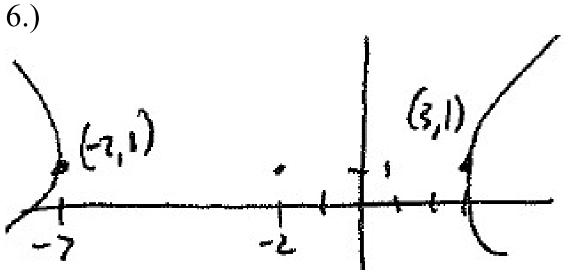
vertices: $(-2,-2)$, $(4,-2)$

conic: hyperbola

4.) CABD

5.) $\frac{(x-2)^2}{16}-\frac{(y-1)^2}{4}=1$

6.)



vertices: $(3,1)$, $(-7,1)$

foci: $(-2+\sqrt{41},1)$, $(-2-\sqrt{41},1)$

7.) $x=\frac{4}{\sqrt{3}}$

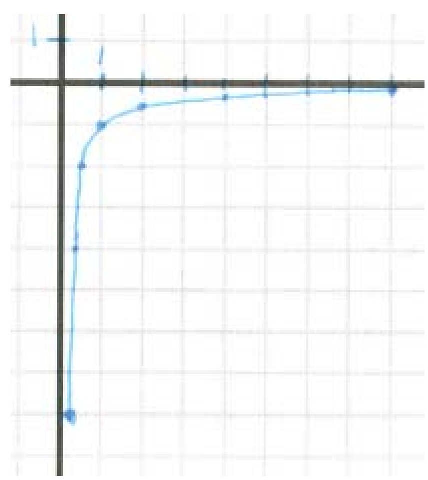
Session 12, Part 1: Parametric Equations

1.) Put into the Pythagorean Identity and get $x^2-y^2=1$.

2.) various answers: $x=2\cos t$; $y=2\sin t$; $t\in[0,2\pi)$

3.)

t	x	y
-3	8	-1/8
-2	4	-1/4
-1	2	-1/2
0	1	-1
1	1/2	-2
2	1/4	-4
3	1/8	-8



4.)

t	x	y
0	-2	-5
4	2	3
m	$\frac{2-{}^{-}2}{4}=1$	$\frac{3-{}^{-}5}{4}=2$

$x = t - 2$

$y = 2t - 5$

$0 \le t \le 4$

5.)

Solve by manipulation

$$\frac{x-4}{6} = \tan^2 t$$

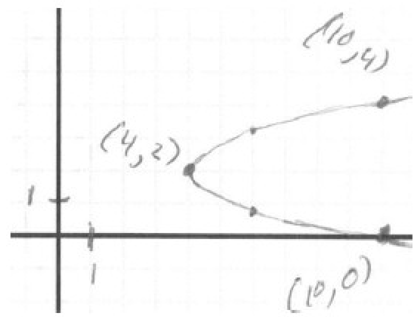
$$\frac{y-2}{2} = \tan t$$

$$\frac{x-4}{6} = \left(\frac{y-2}{2}\right)^2$$

$$\frac{2}{3}(x-4) = (y-2)^2$$

or solve by point plotting

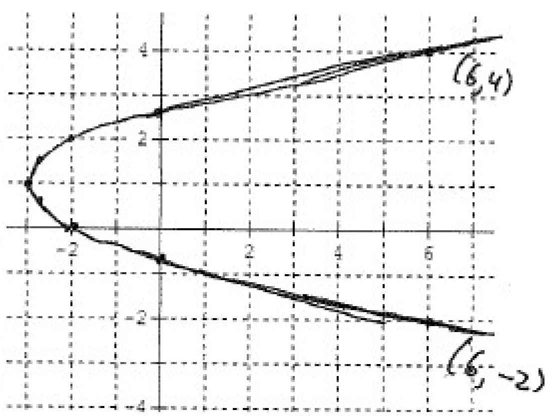
t	x	y
$-\pi/2$	—	—
$-\pi/3$	22	-1.5
$-\pi/4$	10	0
$-\pi/6$	6	0.85
0	4	2
$\pi/6$	6	3.2
$\pi/4$	10	4
$\pi/3$	22	5.5
$\pi/2$	—	—



6.) a, d respectively

7.)

t	x	y
0	-3	1
$\pi/6$	$\frac{4}{3}-\frac{12}{3}=-8/3$ ≈ -2.7	$\frac{1}{\sqrt{3}}+1$ ≈ 1.6
$\pi/4$	-2	2
$\pi/3$	0	$1+\sqrt{3}$ ≈ 2.7
$\pi/2$	undefined	undefined
$2\pi/3$	0	$1-\sqrt{3}$ ≈ -0.7
$3\pi/4$	-2	0
$5\pi/6$	-2.7	$-\frac{1}{\sqrt{3}}+1$ ≈ 0.4
π	-3	1



$$\frac{x+4}{1} = \sec^2 t \qquad y-1 = \tan t$$

$$1 = \sec^2 t - \tan^2 t$$

$$x+4 - (y-1)^2 = 1$$

$$x+3 = (y-1)^2$$