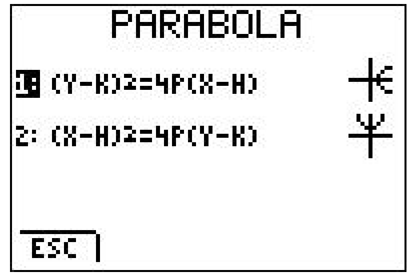
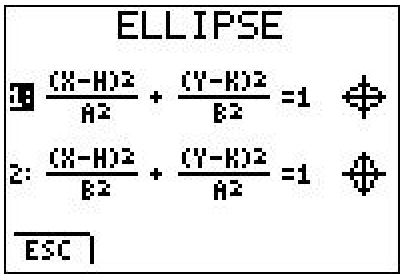
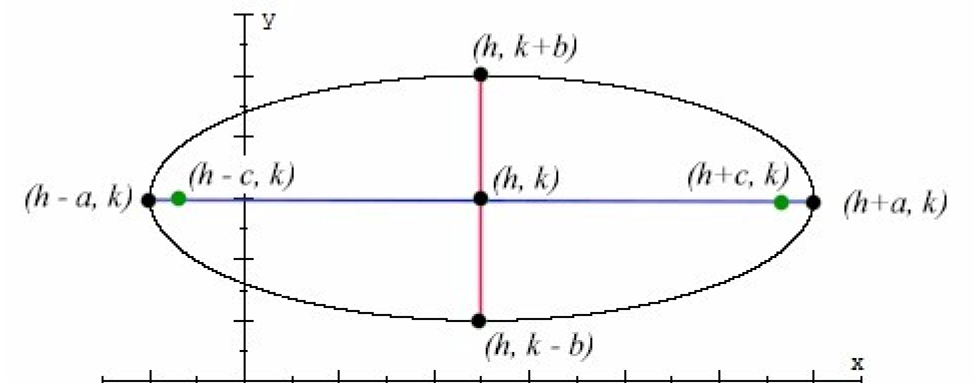
Conics Summary

Parabola: Recall that for the parabola, we let $|p|$ stand for the distance from the vertex to the focal point and the distance from the vertex to the directrix. p is positive when the parabola opens in a positive direction and p is negative when the parabola opens in a negative direction. With this in mind we came up with the standard equations illustrated in this screenshot from the TI-84 calculator, conics application. Note that (h, k) stands for the coordinates of the vertex.

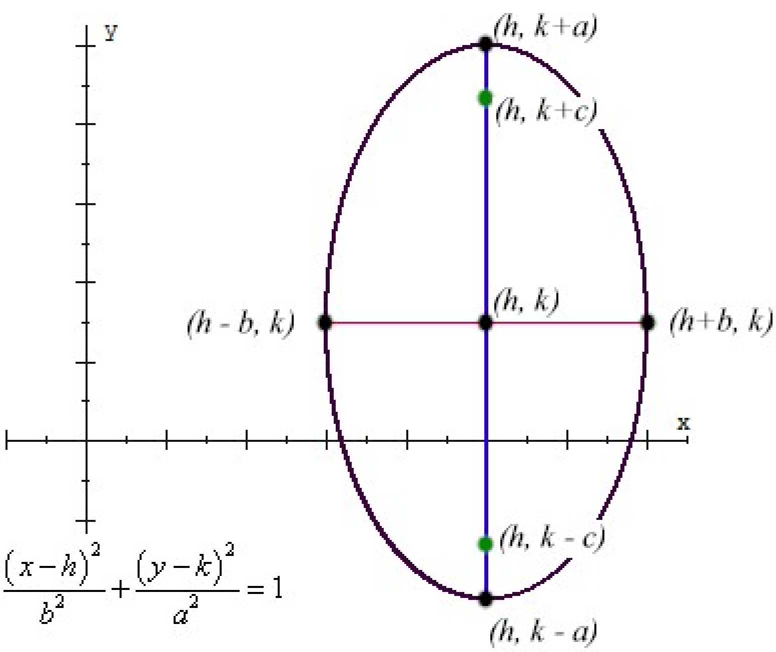




Ellipse: Recall that an ellipse is the set of all points in a plane where the sum of the distances from two fixed points (called foci) is a constant. In this case we let (h, k) stand for the center, c stands for the distance from the center to a focal point, and a stands for the distance from the center to a vertex. In class we demonstrated why the sum constant has to equal $2a$. We also let $2b$ stand for the length of the minor axis. With all this in mind, we came up with:

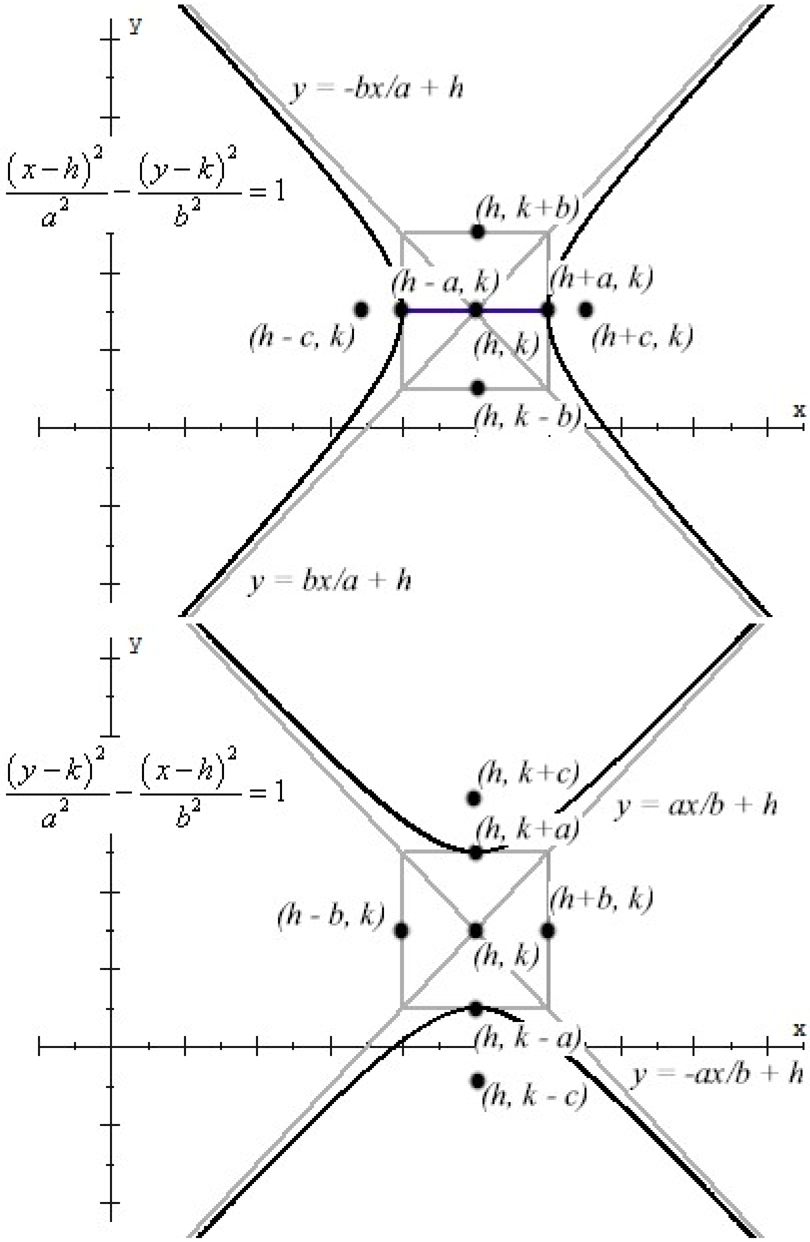


$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Hyperbola: The set of all points in a plane where the difference in the distances from the two foci is a constant. We let (*h, k*) stand for the center, *c* stands for the distance from the center to a focal point, and *a* stands for the distance from the center to a vertex as in the ellipse. The hyperbola has asymptotes that go through its center with slopes ±*b/a* or ±*a/b* depending on the orientation. You can still think of 2*a* as the length of the axis (the distance between the two vertices) and 2*b* as the length of the other axis. In this case *b* helps you draw the asymptotes, which in turn helps you draw the hyperbola.



Exerise: A good exercise is to derive the standard equations of the conics from the definitions. You will want to first estabilish the sum/difference constant in the cases of the hyperbolas and ellipses, by looking at what happens when your arbitrary point is on the vertex. Also, during your derivation process, you will eventually want to get rid of the constant *c*. Do this by letting $c^2 - a^2 = b^2$ for the hyperbola and letting 2*b* be the length of the minor axis in the ellipse.
To see the physical significance of *b* in the hyperbola, you may want to read about the asymptotes of hyperbolas.