Just About Everything a Trig Student Would Want to Know About Vectors

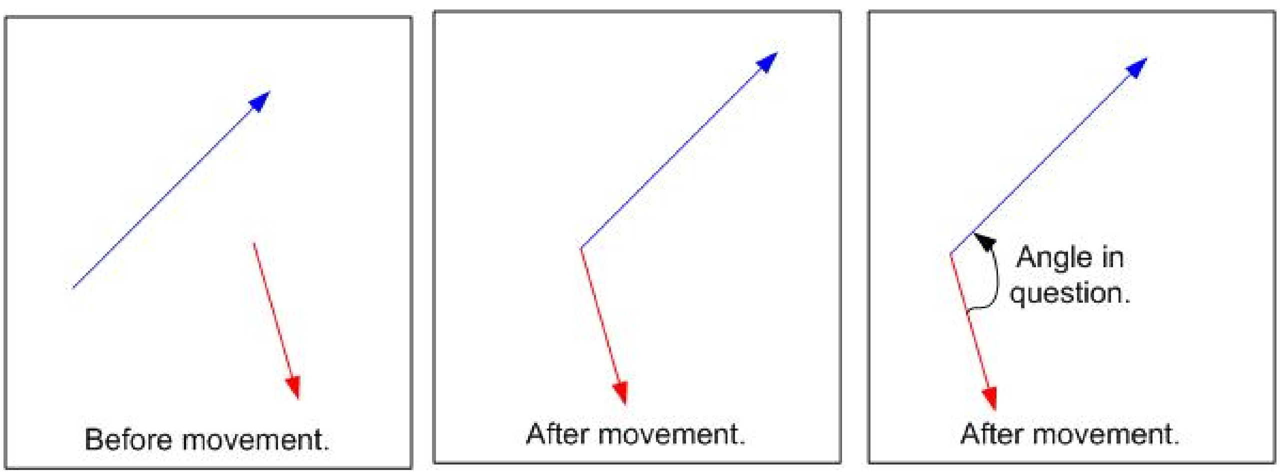
1. A <u>vector</u> is defined by its magnitude and its direction. Thus two vectors are equivalent if they have the same magnitude and same direction.

2. Notational Notes: A vector is indicated by bold type in print and this is equivalent to a handwritten arrow over the vector. A hat is used to indicate a unit vector (a vector of magnitude one). The vector **i** is the unit vector in the positive x direction and the vector **j** is the unit vector in the positive y direction, these would be handwritten as $\hat{i}$ and $\hat{j}$ respectivly. Absolute value notation is used to indicated the magnitude of a vector.

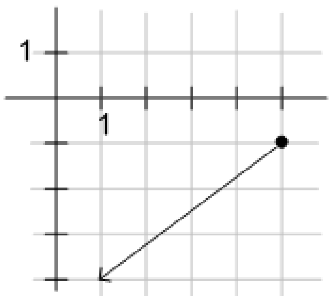
   a. Let **v** be a vector whose magnitude is one. How would you represent **v** when you write it by hand? _________
   b. Suppose the magnitude of of the vector **w** is 3. How would you indicate that by hand? ___________

3. Other Definitions:

   The <u>tail</u> of a vector is the end where the vector starts and the <u>head</u> of a vector is the end where the vector finishes (the end that is pointed to with the arrow). The <u>direction angle</u> of a vector is the angle that a vector makes with the positive *x*-axis when the tail of the vector is placed at the origin. The <u>angle between two vectors</u> is the smallest positive angle you can form with the vectors when the vectors are drawn with their tails together.



Label the tail and head of the vector in this picture. Also draw in the direction angle and label it as θ.

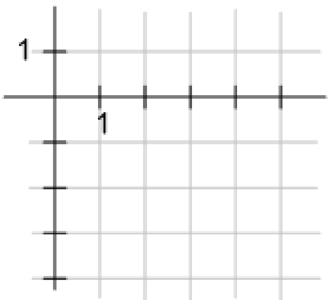


4. Other Notes:
   - A vector can be displayed graphically.
   - A vector can be described in words. For example: where the head and tail are located, or what the vector's magnitude and direction is.
   - A vector can be described by its component parts **u** = *a***i** + *b***j** where *a* and *b* are coefficients of the unit vectors **i** and **j**. See the notes about vector addition for further information.
   - A vector can also be described as a the directed line segment from the origin to a point (*a*, *b*) written: ‹*a*, *b*›.

A.) Which of the following defines a vector? (Circle all that do the job.)

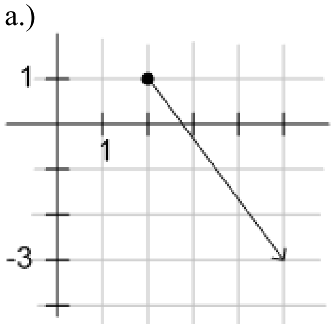
a. A car traveling 45 miles per hour due North.
b. A line segment that connects the point (2, 3) to the point (8, 12).
c. A line segment that goes through the origin at the angle of 65° with the positive *x*-axis.
d. A directed line segment that starts at the point (4, 5) and ends at the point (-3, 7).
e. A 5 inch line segment at an angle of -23° with the positive *x*-axis.

B.) Draw a vector on the following grid that starts at the point (5, 1) and ends at the point (1,-4). Label the vector, as vector **w**.

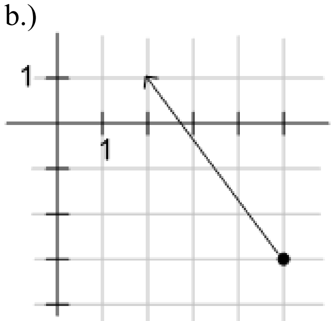
	Fill in the blanks: $\lvert\mathbf{w}\rvert$ = ______ The direction angle of **w** is: __________	Write **w** as $a\mathbf{i} + b\mathbf{j}$.	Write **w** in $\langle a, b\rangle$. notation.

5. Example: Let $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ (which can also be written as: $\langle 3, -4\rangle$ and **u** could also be written as $\vec{u}$). Which of the following vectors are equivalent to **u**?

a.)



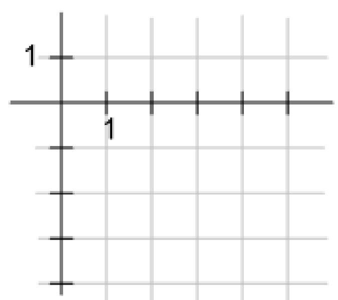
b.)



c.) The vector whose tail is at the origin and whose head is $(5, \tan^{-1}(-4/3))$ (in polar coordinates).

d.) The vector whose tail is at the origin and whose head is $(-5, \tan^{-1}(-4/3))$ (also in polar coordinates)?

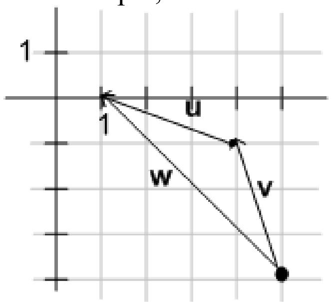
Draw **u** on the following grid with its tail at the origin.



6. Resolving a Vector into its Components: Usually when we resolve a vector into its components we are talking about into its vertical and horizontal components. For example: Let the vector **z** be a vector of magnitude 7 with a direction angle of 115°. Then when you write **z** as $a\mathbf{i} + b\mathbf{j}$ or as $\langle a, b\rangle$, you have resolved **z** into its vertical and horizontal components. $\mathbf{z}=\langle 7\cos 115°, 7\sin 115°\rangle \approx -2.96\mathbf{i} + 6.34\mathbf{j}$ (see notes on vector addition next).

   Let **d** be a vector where $\lvert\mathbf{d}\rvert=12$ and the direction angle of **d** is 75°. Resolve **d** into its components and write it in one of the two ways mentioned in the notes accurate to the hundredths place.

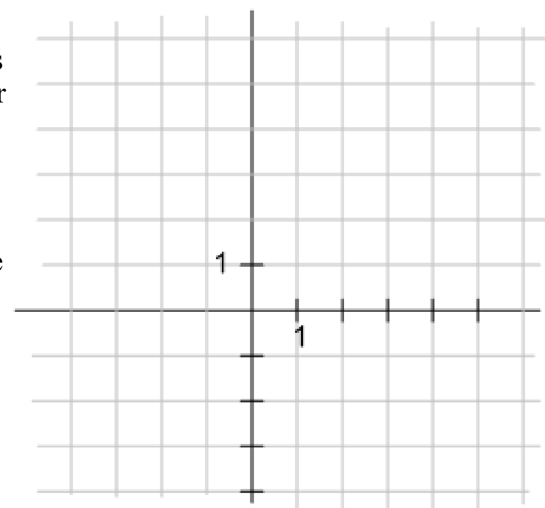
7. Vector Addition: Two vectors are added graphically by placing the tail of one vector on the head of another vector. For example, look at the image where **u** + **v** = **w**.



Resolve each of the vectors **u**, **v**, and **w** in terms of their horizontal and vertical components and then show how you would add **u** and **v** without the picture.

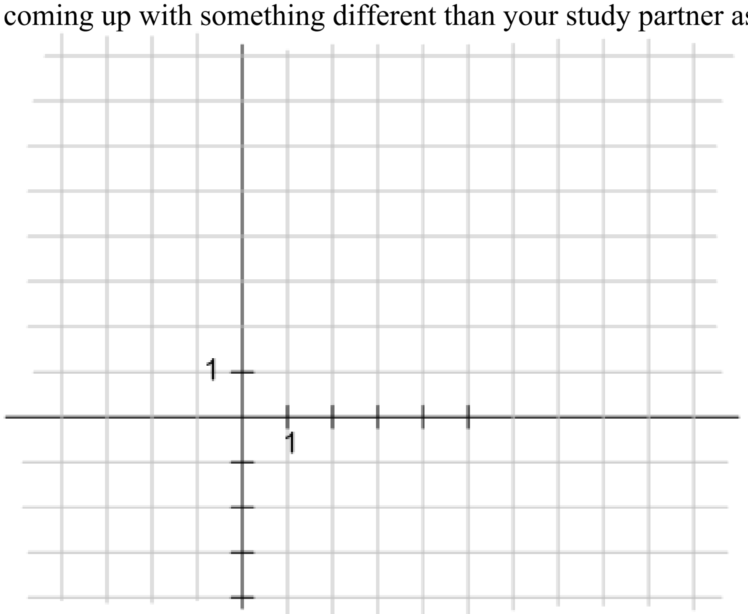
8. Scaler multiplication: Since multiplication is by definition repeated addition, it only makes since that mulitplying by a scaler, *a* (a scaler is a number that is not a vector) would mean that you are adding a vector to itself *a* times. Since the direction doesn't change, this has the net affect of stretching the vector by a factor of *a*.

   Let **p**=‹2, 1›. Write 3**p** in terms of **i** and **j** and then draw both **p** and 3**p** on the same grid. If you have two different colors to write with, please use one color for **p** and another color for 3**p**.



9. Linear Combination: A vector is a <u>linear combination</u> of other vectors if it can be written in terms of the other vectors. The most common examples you see in the book are examples of vectors written in terms of **i** and **j** such as **u** in number 5 above. However, vectors could also be written in terms of other vectors that aren't just simple perpendicular unit vectors. Let **u**=‹1, 1› and **v**=‹2, -1›. Let **w**=4**u**+2**v**. Then from what we know about scaler multiplication and vector addition we know that **w** = ...

   Finish the thought above: **w**=________________
   and then plot **w** on the grid below. Next, show how **w**=4**u**+2**v** graphically. There are 15 different ways to show this, so make sure that you do it in a way that is different than anybody you are working with. If you have troubles coming up with something different than your study partner ask that student for help.

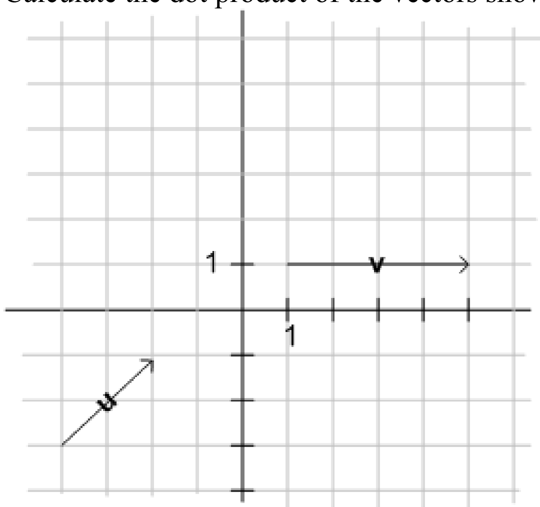


10. Dot Product: There are two types of vector multiplication. I am talking about multiplication between two vectors as opposed to between a vector and a scaler as above. We are only going to study one of them in this class, dot product. The dot product of two vectors, **u**=$\langle u_1, u_2\rangle$ and **v**=$\langle v_1, v_2\rangle$ can be defined two different but equivalent ways:

1. **u•v**=|**u**||**v**|cos θ, where θ is the angle between the vectors when they are drawn with their tails at the same location.
2. **u•v** = $u_1 v_1 + u_2 v_2$.

Notice that the right hand side of both equations is a scaler, not a vector. Two prove that these two definitions are equivalent all you need to do is draw the vectors with their tails together, make a triangle by connecting their heads with a line segment, and then use the Law of Cosines. You'll want to consider the case where the angle between the vectors is zero or π separately.

Calculate the dot product of the vectors shown on the following graph both of the ways named in the lecture notes.



11. Applying the Dot Product to Find the Angles Between Two Vectors: Since the dot product can be calculated two different ways, it can be used to easily find the angle between two vectors. Whenever you are asked to find the angle between two vectors, you want to find the smaller angle that would be between the two vectors if you were to place the vectors tails together as in the following images. Recall that you can move a vector around and it is still considered the same vector.

Let **u**=6**i** - 3**j** and **v**=-4**i** -**j**. Set the right sides of both of the equations for dot product in the lecture notes equal to each other and solve for cos θ and then use arccos of your answer to find θ.

12. **Finding Magnitude and Direction of a Sum of Vectors**

Resolving a vector into components is actually using the vector as the hypotenuse of a triangle and the components as the legs of a right triangle, then finding the legs of the triangle using the definitions of sine and cosine. If the components are to be parallel to the axes, it is often easiest to move the vector temporarily so that its initial point (tail) is at the origin and its direction is preserved. Then the x- and y-components are the x- and y-coordinates of its terminal point (head).

Example 1:

Resolve the vector representing the flight of a plane at 200 miles per hour at a heading of 300 degrees into its x- and y-components.

Example 2:

Resolve a wind of 40 miles per hour from a heading of 64° into x- and y-components.

Example 3:

Find the actual direction of travel and ground speed of a plane flying at 200 miles per hour at a heading of 300 degrees if there is a wind of 40 miles per hour blowing <u>from</u> a heading of 64°.

We already have the components of the separate vectors, so we can add them (by adding corresponding components).

This method avoids having to draw the parallelogram and the resultant vector, then calculate one of the angles in the triangle formed by the original vectors and the resultant, then use the law of cosines and law of sines. The price is having to resolve the given vectors into components and, at the end of the problem, find the magnitude and direction of the sum which is written in component form. Either method can be used, depending on one's preference.