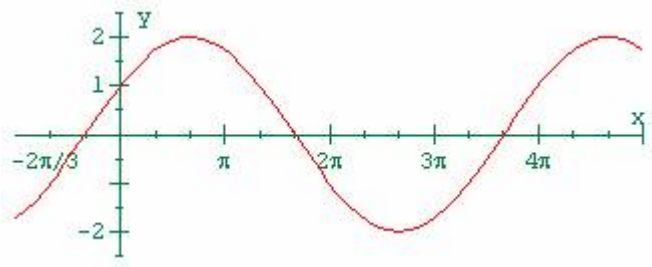
MATH122
Writing the equation for a given trig graph



The graph above could be that of either a sine or cosine function, depending on whether one wavelength is considered as on the interval [-π/3, 3π+2π/3] or [2π/3, 4π+2π/3] respectively.

Example 1: Write an equation for the graph in the form $y = A \sin (Bx + C)$.

The amplitude of the graph is 2, so $A = 2$.

We know that one typical complete wave of the sine function starts at a y-value of 0, increases to 1, decreases through 0 and on to –1, and then increases to a y-value of 0, so the interval [-π/3, 3π +2π/3] contains one wave length of the function. The length of the interval tells us that the period is (3π +2π/3) - (-π/3) = 4π.

Using the formula: period = $\frac{2\pi}{B}$, we set $4\pi = \frac{2\pi}{B}$ and solve for B, finding that $B = \frac{1}{2}$.

We have already observed from the graph that the phase shift is $-\frac{\pi}{3}$, so we can use the formula:

phase shift = $-\frac{C}{B}$, set $-\frac{\pi}{3} = -\frac{C}{\frac{1}{2}}$ and solve for C, finding that $C = \frac{\pi}{6}$.

The equation of the graph is $y = 2\sin\left(\frac{1}{2}x + \frac{\pi}{6}\right)$

Example 2: Write an equation for the same graph in the form $y = A\cos(Bx + C)$.

The amplitude of the graph is still 2, so $A = 2$.

We know that one typical complete wave of the cosine function starts at a y-value of 1, decreases through 0 to –1, and then increases through 0 to 1, so the interval [2 p /3, 4 p +2 p /3] contains one wave length of the function. The length of the interval tells us that the period is 4 p +2 p /3 - 2 p /3 = 4 p , the same period as we found in Example 1, so $B = \frac{1}{2}$ as before.

We can use a phase shift of $\frac{2\pi}{3}$, so the formula: phase shift = $-\frac{C}{B}$ becomes $\frac{2\pi}{3} = -\frac{C}{\frac{1}{2}}$ and $C = -\frac{\pi}{3}$.

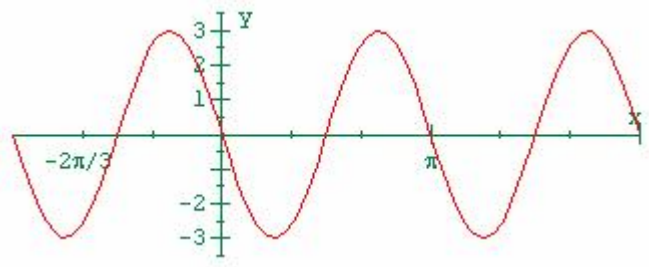
The equation of the graph is also $y = 2\cos\left(\frac{1}{2}x - \frac{\pi}{3}\right)$.

You may want to graph the equations for Example 1 and Example 2 to see whether the graphs are really the same.

Problems:

Write the equation of the graph in the form $y = A \sin (Bx + C)$.

1.



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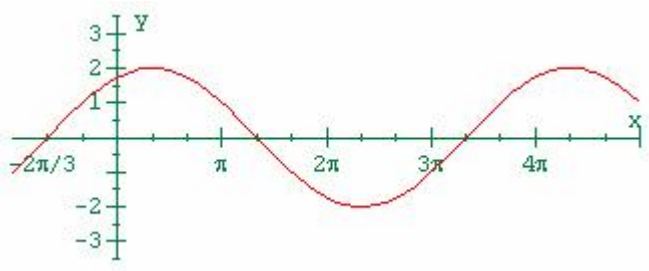

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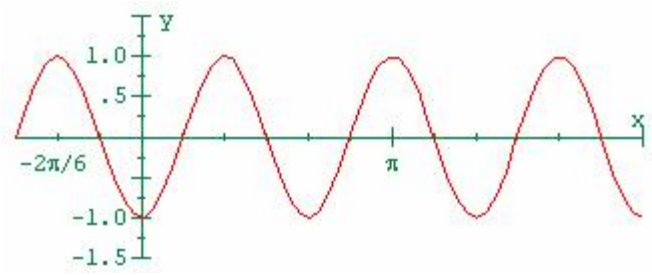
Write the equation of the graph in the form $y = A \cos (Bx + C)$.

3.



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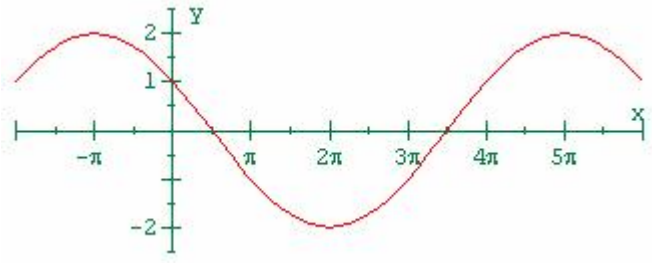

Answers: (Note: answers are not unique; the certain way to check them is to sketch a graph or use your calculator to see if the graph is the same as the one given)

1. $y = 3\sin(2x + \pi)$ 2. $y = 2\sin\left(\frac{1}{2}x + \frac{\pi}{3}\right)$

3. $y = \cos(3x - \pi)$ 4. $y = 2\cos\left(\frac{1}{3}x + \frac{\pi}{3}\right)$