Math 122 Final Review Guide

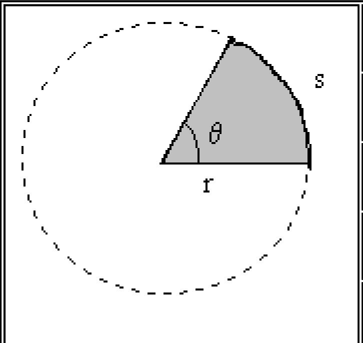
Some questions are a combination of ideas from more than one section. Listed are the main sections that a question relates to.

5.4 1. Convert 97° to radians.

5.3 2. If 1035° is in standard position, find the smallest positive coterminal angle.

5.3 3. If $\theta = 500^R$ is in standard position, find the smallest positive coterminal angle.

5.4 4. Fill in the following chart:



s	r	θ	Area
	3	20°	
10		2	
20	4		
		π	9π/2

5.3 5. Let $P(t) = \left(\frac{8}{17}, \frac{15}{17}\right)$ be a point on the standard unit circle corresponding to the angle *t* measured in radians from the positive *x*-axis. Find:

a) P(-*t*)

b) P(*t* + π/2)

c) P(*t* - π/2)

d) P(π - *t*)

e) P(t - π)

6. Evaluate each of the following exactly.

5.3 a) cos(5π/4)

5.3 b) tan(5π/3)

6.2 c) sin 67.5° Hint: 135°/2 = 67.5°

6.1 d) csc 15° Hint: 15° = 45° - 30°

6.4 e) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

6.4 f) $\cos\left(2\tan^{-1}\left(\frac{12}{5}\right)\right)$

6.2 g) $\sin(2x)$ if $\pi/2 < x < \pi$ and $\cos x = -3/5$

6.2 h) $\sin(x/2)$ if $0 < x < \pi/2$ and $\sin x = \frac{\sqrt{5}}{3}$

6.4 i) $\sin^{-1}(\sin 1.2)$

6.4 j) $\sin^{-1}(\sin 1.6)$

6.4 k) $\sin(\sin^{-1} 0.9)$

7. Find the exact values of the trigonometric functions of t for each of the following given conditions:

5.3 a) $\csc t = 3$ and $\cot t < 0$

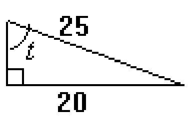
5.3 b) $\cot t = -5/12$ and $\cos t > 0$

5.3 c) The point (8, 15) is on the terminal side and t is in standard position.

5.3 d) The initial side of t is along the positive x-axis and the terminal side of t is in quadrant IV along the line $3x + 2y = 5$.

5.1 e) $\cos t = 7/25$ and t is an acute angle

5.1 f)



10.1 8. Express 3 + 8 + 13 + 18 + 23 in terms of summation notation.

6.3 9. Verify the following identities by working each side independently:

a) $\sec x \csc x = 2 \csc 2x$

b) $\sin 3x = 3 \sin x - 4 \sin^3 x$ (Hint: $3x = 2x + x$)

10. The following questions are asking what a trig functions value approaches as its angle t approaches the indicated value. Fill in the blank for each of the following:

5.5 a) $\tan t \to$ ____ as $t \to (\pi/2)^+$

5.5 b) $\cot t \to$ ____ as $t \to (\pi/2)^+$

6.4 c) $\tan^{-1} t \to$ ____ as $t \to -\infty$

6.4 d) $\sin^{-1} t \to$ ____ as $t \to 1^-$

5.5 e) $\sin t \to$ ____ as $t \to (3\pi/2)^-$

5.5 f) cos $t \to$ ____ as $t \to (3\pi/2)^-$

5.5 g) sec $t \to$ ____ as $t \to (3\pi/2)^-$

5.5 h) csc $t \to$ ____ as $t \to (3\pi/2)^-$

6.5 11. For each of the following equations find all **exact** solutions for x in the interval $[0, 4\pi]$.

a) $\cos x = -1$

b) $\sin x = \frac{\sqrt{3}}{2}$

c) $\sec x = \sqrt{2}$

d) $\cot x = 1$

e) $\cos^2 x + 2\sin x = 1$

f) $\cos 2x = \sin x$

g) $2\sin 3x + 1 = 0$

h) $\sec^2 x = 2\tan x$

6.5 12. Find all solutions for x in each of the following equations.

a) $2\cos^2 x + 3\cos x + 1 = 0$

b) $4\sin^2 x - 3 = 0$

c) $\cos(2x + \pi/6) = 1/2$

d) $\cos 2x + \sin x = 0$

5.6 13. Sketch the graph of each of the following. Label at least 3 points exactly. Draw a dotted boundary graph where appropriate.

a) $y = |\tan x|$

b) $y = \left(\frac{1}{2}\right)^x \sin x$

c) $y = \left|\frac{x}{2}\right| \cos x$

d) $y = \cos x + \sin(2x)$

7.1&7.2 14. Solve the following triangles for all possible solutions.

a) A = 90° , B = 30° , c = 6

b) a = 300, B = 40° , C = 35°

c) C = 90° , a = $2\sqrt{3}$, c = 4

d) C = 90° , b = $3\sqrt{2}$, c = 6

e) a = 11.2, c = 9.3, C = 53° 20'

f) A = 127° , a = 26, b = 16

g) a = 12.7, b = 22.1, C = 58.5°

h) B = 25° , b = 2.6, c = 7

i) a = 12, b = 20, c =25

j) a = 5, b = 7, c = 13

5.6 15. A high point on a curve that can be described by the function $y = a\,sin(bx + c) + d$ is located at (5, 7) and a low point is located at (1, -1). Each period is 8 units long. Find a, b, c, and d.

6.4 16. Write as an algebraic expression in x for $x > 0$:

a) $\tan\left(\sec^{-1}\frac{\sqrt{x^2+9}}{x}\right)$

b) $\cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{x}\right)\right)$

8.2 17. Solve the following system of equations:

$$\begin{aligned} 2x + 3y - 6z &= 2 \\ 6x - y + 3z &= 3 \\ 10x + 2y - 15z &= 2 \end{aligned}$$

10.1 18. Evaluate $\sum_{n=1}^{100}(n-2)(n-3)$ using the fact that $\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$.

10.1 19. Evaluate $\sum_{n=1}^{4}\frac{(-1)^{n+1}}{n}$

10.1 20. Write out the first four terms and the eighth term of the sequence whose nth term is $\frac{2n+3}{2n-3}$.

10.2 21. Find a_{30} in the arithmetic progression where $a_4 = -5$ and $a_{23} = 52$.

10.2&.3 22. The following sequences are either arithmetic or geometric:

$\{a_n\}$ = 30, 34, 38, ... $\{b_n\}$ = 80, -120, 180, -270, ...

a) Find the recursive formula for both sequences.

b) Find the explicit formula for both sequences.

c) Find a_{10} and b_{10}.

10.7 23. Use the binomial expansion of $\left(x - \frac{3}{y}\right)^{12}$ to

a. find the seventh term.
b. find the coefficient of x^3/y^9.

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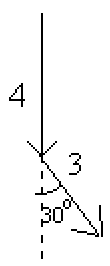
7.5 & 7.6 24. A current is flowing downstream in the middle of a mile wide river at a rate of 4 miles per hour due South. Noelle rows a boat 30° East of South to get to Kevin who fell overboard.

	a. If Noelle rows at a rate of 3 miles per hour in still water, what is her resultant speed and direction? b. What are the Easternly and Southernly components of Noelle's resultant velocity?

7.5 & 7.6 25. Let vector **m** have magnitude 4 and angle 60° and vector **n** have magnitude 6 and angle -30°.

a. Evaluate **m** + **n** and write your answer in the form $a\mathbf{i} + b\mathbf{j}$
b. Evaluate |**m** + **n**| exactly.
c. Find the direction of **m** + **n** rounded to the nearest hundredth of a degree.

5.6 26. Match the following equations to their graphs by placing the letter of the graph next to the corresponding equation.

(i)____ $y = \cos x$ $\qquad$ (ii)____ $y = 2\cos x$

(iii)____ $y = \cos 2x$ $\qquad$ (iv)____ $y = \cos \pi x + 1$

(v)____ $y = \cos(\pi x + \pi)$ $\qquad$ (vi)____ $y = \cos(2x + \pi)$

(vii)____ $y = \tan x$ $\qquad$ $(viii)$____ $y = 2\tan x$

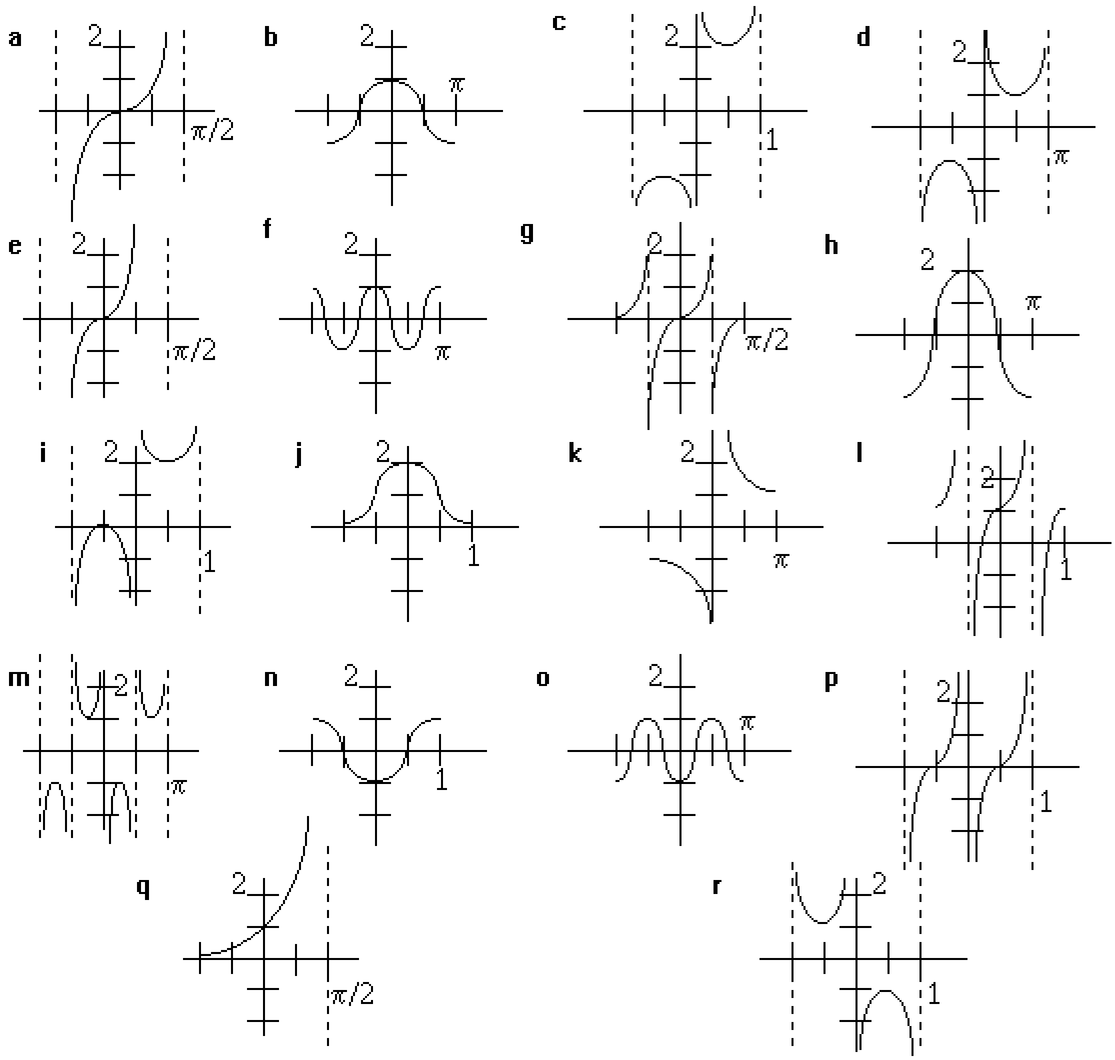
(ix)____ $y = \tan 2x$ $\qquad$ (x)____ $y = \tan \pi x + 1$

(xi)____ $y = \tan(\pi x + \frac{\pi}{2})$ $\qquad$ (xii)____ $y = \tan(\frac{x}{2} + \pi)$

$(xiii)$____ $y = \csc x$ $\qquad$ (xiv)____ $y = 2\csc x$

(xv)____ $y = \csc \frac{x}{2}$ $\qquad$ (xvi)____ $y = \csc \pi x + 1$

$(xvii)$____ $y = \csc(\pi x + \pi)$ $\qquad$ $(xviii)$____ $y = \csc(2x + \pi)$

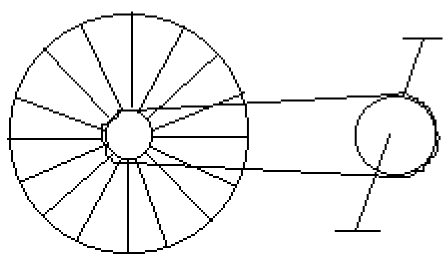


7.5 & 7.6 27. Graph $y = 4\sin\left(\frac{\pi}{2}x - \frac{\pi}{8}\right) + 1$. Use a scale that makes sense for the problem and draw the graph carefully through exact points on the grid where ever possible. Draw at least two full cycles.

5.4 28. A child is riding his bicycle on flat land. The diameter of his tires is 20". He is riding at a rate of 10 miles per hour.

a.) How many revolutions per minute is his tire making?

b) If the radius of the sprocket that his tire is attached to is 2 inches and the radius of the sprocket that his pedals are attached to is 4 inches, how fast is he peddling (in revolutions per minute)?



5.2 29. How tall is a tree if the angle between the ground and the tree top is 45° at a distance of 200 feet from the base of the tree?

10.3 30. If a ball falls from an altitude of 200 feet and has the property that it rebounds to three-quarters of its original altitude after each fall, how far has it traveled by the time it hits the ground for the tenth time? What is the total distance it travels?

5.2 31. If a 30-foot ladder just reaches a roof 20 feet high, what angle does the ladder make with the ground?

7.1&7.2 or 7.6 32. If a ship travels straight east for 2½ hours at 100 nautical miles per hour and then veers 45° north of east for 1½ hours at the same rate of speed, how far is the ship from its starting point?

7.1&7.2 33. A triangular plot of land has sides of 100 ft., 120 ft., and 135 ft.. What is the area of the land?

8.2 34. Dick, Jane, and Sally have each made punch. Dick's punch has no fruitjuice. Jane's is 1% juice by volume and Sally's is 7% juice by volume. Dick adds twice as much of his punch to Sally's to dilute hers. Jane pours in some of her punch. Now they have 5 gallons of punch that is 1½ % juice. How much of each was used?

5.2 35. A tower 80 meters high stands on a cliff. From the top and bottom of the tower, the angles of depression to a ship are 18° and 14° respectively.

a) What is the distance of the ship from the foot of the tower?

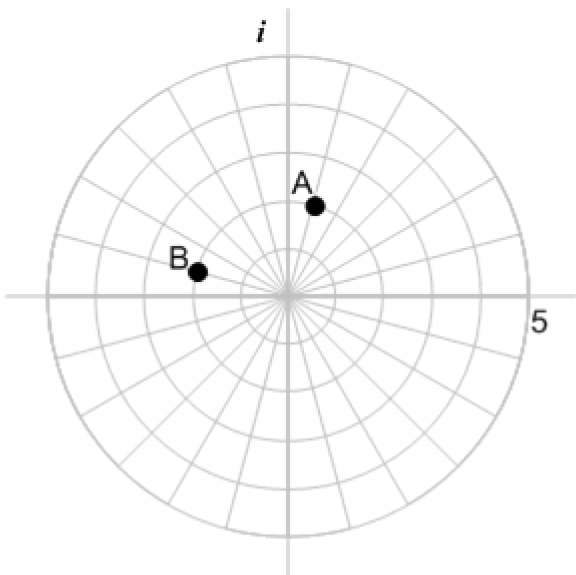
b) What is the distance of the ship from the foot of the cliff?

c) How high is the cliff?

5.2 36. The furthest point of the earth's surface visible from a mountain top is 80 miles from that mountain top. Find the height of the mountain if the earth's diameter is 7912 miles.

7.3 37.

a. Evaluate $\dfrac{6\sqrt{2}\left(\cos\left(\pi/12\right)+i\sin\left(\pi/12\right)\right)}{3-3i}$ exactly. Write your answer both in trigonometric form and in $a + bi$ form.
b. Let complex numbers A and B be as indicated on the graph. Plot C and tell what C is in $a + bi$ form, where C=A×B.



7.4 38.

a. Find the polar coordinates of the points A and B in the graph that goes with number 35 b above.
b. Convert A and B in part a to rectangular coordinates rounded to the thousandths place.
c. Convert $r = 4\cos\theta$ to a rectangular equation, identify the shape, and give details about the shape (for example if you have a parabola tell where its vertex is located and which way it opens and how fat or skinny it is compared to $y = x^2$).
d. Convert $(x - 3)^2 + (y - 4)^2 = 25$ to a polar equation.

6.3 39. A student is trying to prove: $\dfrac{2\cos 2x}{\sin 2x} = \cot x - \tan x$ and makes a mistake. Name the mistake in the following proof.

$$\frac{2\cos 2x}{\sin 2x} = \cot x - \tan x$$
$$2\cos 2x = \sin 2x(\cot x - \tan x)$$
$$2\cos 2x = 2\sin x\cos x\left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\right)$$
$$2\cos 2x = 2\cos^2 x - 2\sin^2 x$$
$$2\cos 2x = 2(\cos^2 x - \sin^2 x)$$

6.3 40. A student is trying to prove: $\frac{\csc t}{\sec t} = \cot t$ and makes a couple of mistakes. Name the mistakes in the following proof.

$$LHS = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \cot x = RHS$$

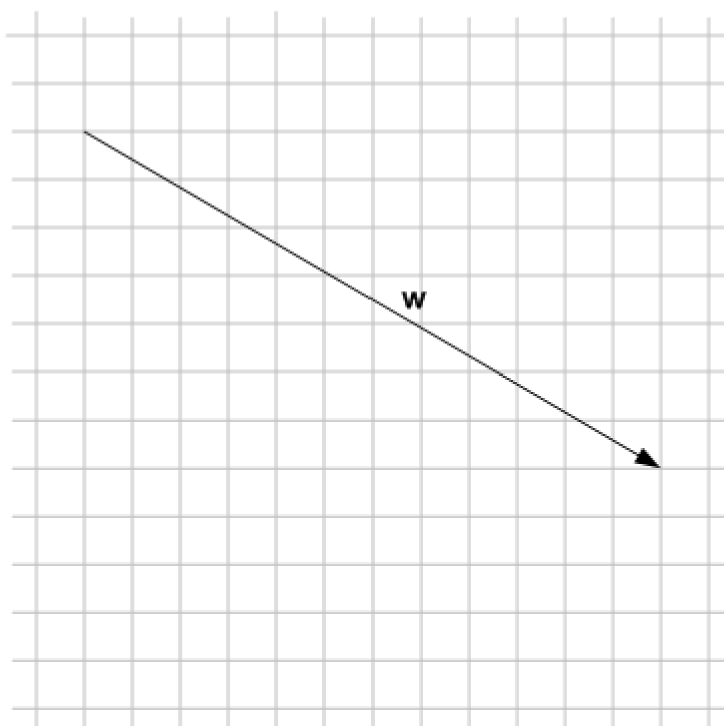
6.3 41. A student is trying to prove: $\cos^2 t - \sin^2 t = 2\cos^2 t - 1$ and makes a couple of mistakes. Name the mistakes in the following proof.

$$\cos^2 t - \sin^2 t = 2\cos^2 t - 1$$
$$1 = \cos 2t$$
$$2t = 2k\pi, \quad k \in Z$$
$$t = k\pi, \quad k \in Z$$

7.5 & 7.6 42.

a. Write **w** from the graph at the right in terms of **i** and **j**.

b. Let **u**=<3,4> and **v**=<2, -5> Draw **w** as a linear combination of **u** and **v**, i.e. as a sum of some **u**'s and **v**'s.



Solutions:

1.) 1.693^R 2.) 315° 3.) 3.628^R

4.)

s	r	θ	Area
π/3	3	20°	π/2
10	5	2	25
20	4	5	40
3π	3	π	9π/2

5.) (a.) $\left(\frac{8}{17}, \frac{-15}{17}\right)$ (b.) $\left(\frac{-15}{17}, \frac{8}{17}\right)$ (c.) $\left(\frac{15}{17}, \frac{-8}{17}\right)$ (d.) $\left(\frac{-8}{17}, \frac{15}{17}\right)$ (e.) $\left(\frac{-8}{17}, \frac{-15}{17}\right)$

6.) (a.) $\frac{-\sqrt{2}}{2}$ (b.) $-\sqrt{3}$ (c.) $\frac{\sqrt{2+\sqrt{2}}}{2}$ (d.) $\sqrt{6}+\sqrt{2}$ (e.) -π /3 (f.) -119/169 (g.) -24/25 (h.) $1/\sqrt{6}$ (i.) 1.2 (j.) π - 1.6 (k.) 0.9

7.)

	(a)	(b)	(c)	(d)	(e)	(f)
cos *t*	$-2\sqrt{2}/3$	5/13	8/17	$2/\sqrt{13}$	7/25	3/5
sin *t*	1/3	-12/13	15/17	$-3/\sqrt{13}$	24/25	4/5
tan *t*	$-1/2\sqrt{2}$	-12/5	15/8	-3/2	24/7	4/3
sec *t*	$-3/2\sqrt{2}$	13/5	17/8	$\sqrt{13}/2$	25/7	5/3
csc *t*	3	-13/12	17/15	$-\sqrt{13}/3$	25/24	5/4
cot *t*	$-2\sqrt{2}$	-5/12	8/15	-2/3	7/24	3/4

8.) $\sum_{i=1}^{5}(5i-2)$

9.) (a.)

(b.)

LHS = $\sin 3x = \sin(2x + x)$

$= \sin 2x \cos x + \sin x \cos 2x$

$$\sec x \csc x = 2\csc 2x$$
$$\frac{1}{\cos x}\cdot\frac{1}{\sin x} = \frac{2}{\sin 2x}$$
$$\frac{1}{\cos x \sin x} = \frac{2}{2\sin x \cos x}$$
$$\frac{1}{\cos x \sin x} = \frac{1}{\sin x \cos x}$$

$= 2\sin x \cos x \cos x + \sin x(1 - 2\sin^2 x)$

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$ = RHS

10.) (a.) $-\infty$, (b.) 0, (c.) $-\pi/2$, (d.) $\pi/2$, (e.) -1 (f.) 0, (g.) $-\infty$, (h.) -1

11.) (a.) π , 3π (b.) $\pi/3$, $2\pi/3$, $7\pi/3$, $8\pi/3$ (c.) $\pi/4$, $7\pi/4$, $9\pi/4$, $15\pi/4$ (d.) $\pi/4$, $5\pi/4$, $9\pi/4$, $13\pi/4$ (e.) 0, π , 2π , 3π , 4π (f.) $\pi/6$, $5\pi/6$, $3\pi/2$, $13\pi/6$, $17\pi/6$, $7\pi/2$ (g.) $7\pi/18$, $11\pi/18$, $19\pi/18$, $23\pi/18$, $31\pi/18$, $35\pi/18$, $43\pi/18$, $47\pi/18$, $55\pi/18$, $59\pi/18$, $67\pi/18$, $71\pi/18$ (h) $\pi/4$, $5\pi/4$, $9\pi/4$, $13\pi/4$

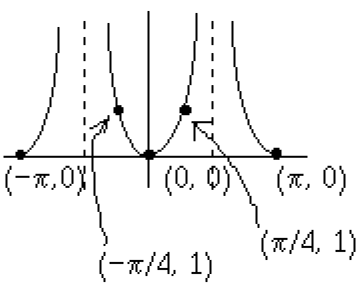
12.) a.) $\pm\frac{2\pi}{3} + 2k\pi,\ \pi + 2k\pi$, k any integer

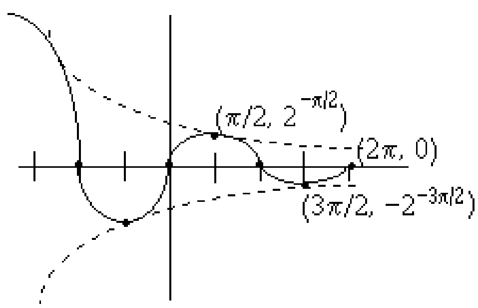
b.) $\pm\frac{\pi}{3} + 2k\pi, \pm\frac{2\pi}{3} + 2k\pi$, k any integer

c.) $\frac{\pi}{12} + k\pi, \frac{-\pi}{4} + k\pi$, k any integer

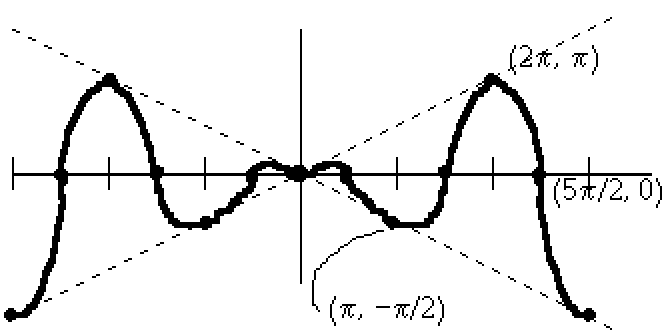
d.) $\frac{-\pi}{6} + 2k\pi, \frac{-5\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi$, k any integer

13.) a.) b.)

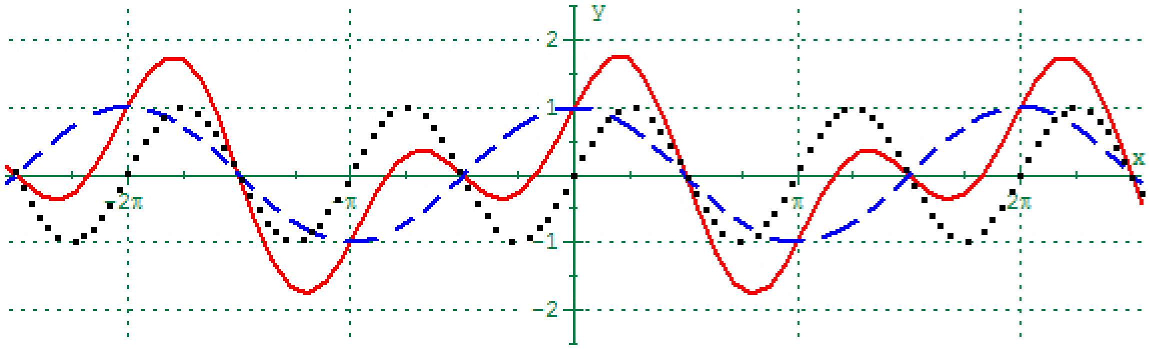




c.)



d.)The solid red line represents the answer. The dotted lines are the curves that were added to achieve the answer.



14.) a.) C = 60° , a = $4\sqrt{3}$, b = $2\sqrt{3}$ b.) A = 105° , b = 199.639, c = 178.143 c.) A = 60°, B = 30° , b = 2 d.) A = 45° , B = 45° , a = $3\sqrt{2}$

e.) two solutions: A = 104.98° , B = 21.68° , b = 4.284 A = 75.02° , B = 51.65° , b = 9.093

f.) B = 29.437° , C = 23.563° , c = 13.014 g.) A = 35.001° , B = 86.499° , c = 18.874 h.) no solution

i.) A = 28.237° , B = 52.048° , C = 94.715° j.) no solution

15.) a = 4, b = π/4, c = -3π/4, d = 3

16.) a.) 3/*x* b.) $\sqrt{\frac{1+x}{2x}}$

17.) *x* = ½, *y* = 1, *z* = $\frac{1}{3}$ 18.) 313,700 19.) $\frac{7}{12}$

20.) -5, 7, 3, $\frac{11}{5}$, …, $\frac{19}{13}$ ← *eigth term* 21.) $a_{30} = 73$

22.) a.) $a_n = a_{n-1} + 4, a_1 = 30; \quad b_n = \frac{-3}{2} b_{n-1}, b_1 = 80$ b.) $a_n = 30 + 4(n-1), \quad b_n = 80\left(\frac{-3}{2}\right)^{n-1}$

c.) $a_{10} = 66, \quad b_{10} = \dfrac{-98415}{32}$

23.) a.) $673{,}596\dfrac{x^6}{y^6}$ b.) -43,302,600

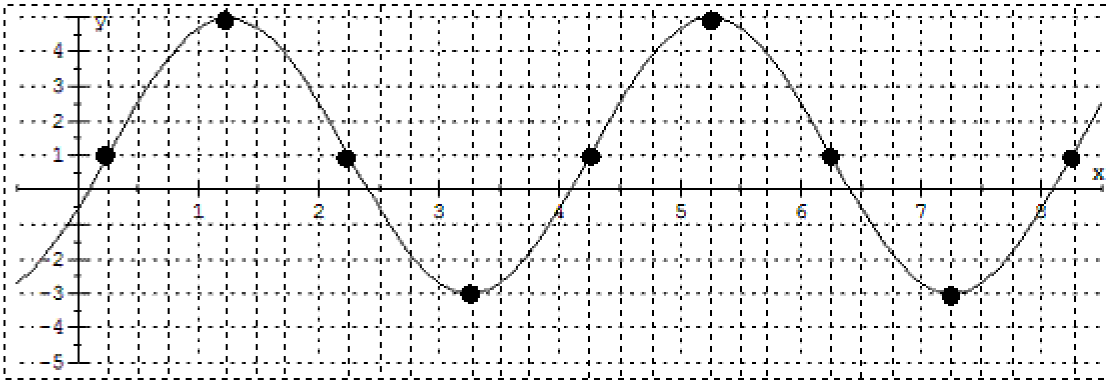
24.) (a.) 12.81° East of South, 6.77 miles per hour (b.) 6.60 mph South and 1.50 mph East

25.) a. $\left(2+3\sqrt{3}\right)\hat{\mathbf{i}} + \left(2\sqrt{3}-3\right)\hat{\mathbf{j}}$ b. $\sqrt{37-4\sqrt{3}}$ c. 3.69°

26.)

(i) b (ii) h (iii) f (iv) j (v) n (vi) o (vii) a (viii) e (ix) g (x) l (xi) p (xii) q (xiii) d (xiv) c (xv) k (xvi) i (xvii) r (xviii) m

27.) Note: I've marked the points that you should get exactly right.



28.) (a) 168.07 rev/min (b) 264/π ≈ 84 rev/min

29.) 200 ft

30.) 1309.898 ft

31.) 41.81°

32.) 371.53 nautical miles

33.) 5798 sq. feet

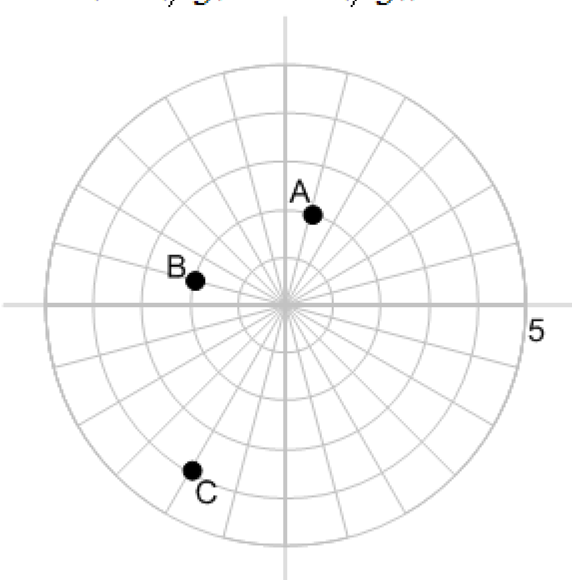
34.) Sally's .625 gal, Jane's 3.125 gal, Dick's 1.25 gal

35.) a.) 1090.716 m, b.) 1058.317 m c.) 263,368 m

36.) 0.809 miles

37.) a.) $2\left(\cos\left(\pi/3\right)+i\sin\left(\pi/3\right)\right) = 1+\sqrt{3}i$

b.) C= -2 - 2√(3)***i***



38.) a.) A=(2, 75°), B=(2, 165°)
b.) A=(0.518, 1.932), B=(-1.932, 0.518)

c.) $(x - 2)^2 + y^2 = 4$ Circle centered at (2, 0) with a radius of 2.
d.) $r = 6\cos\theta + 8\sin\theta$

39.) The student multiplied through by sin $2x$. You are suppose to work both sides of a proof independently.

40.) The student mixed up cosecant and secant. Sine/cosine does not equal cotangent.

41.) $\cos^2 t - \sin^2 t$ does not equal 1. The student started solving for t instead of proving the identity.

42. a.) **w**=12**i** -7**j**
b.)**w**=3**v**+2**u** so there are 10 ways you could represent this on the graph. I've shown one way here.